Update on Squeezed Hadronic Correlations @ RHIC Energies

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WPCF 2010 - Kiev

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50 IFT anos

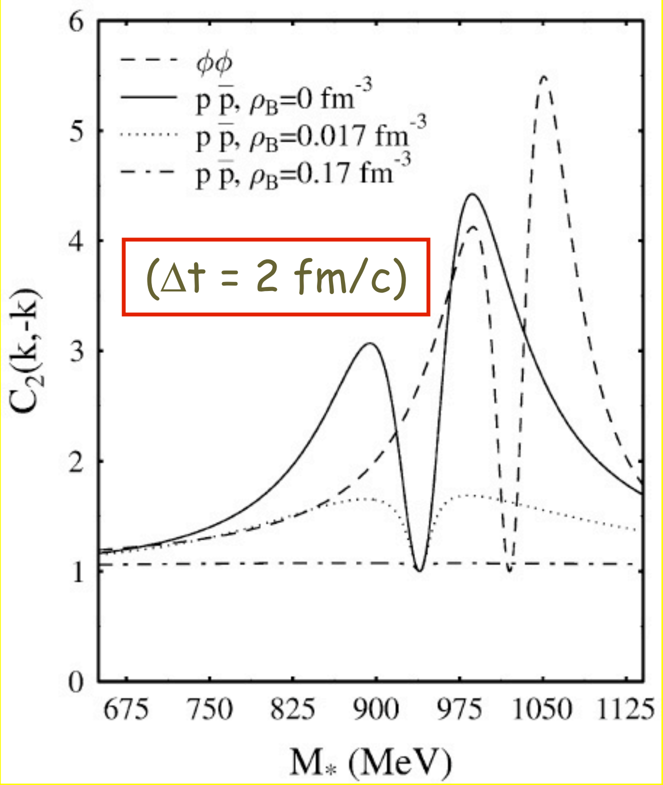
Outline

- Brief review on squeezed correlations
- The effects of time emission parametrization on squeezing: instantaneous, Lorentzian & Lévy distributions
- Predictions of the model
- Comparison with first preliminary data (PHENIX, WPCF 2009)

About a decade ago…

- Late 90's: Back-to-Back Correlations (BBC) among boson-antiboson pairs → shown to exist if the masses of the particles were modified in a hot and dense medium [Asakawa, Csörgő & Gyulassy, P.R.L. 83 (1999) 4013]

- Shortly after → similar BBC shown to exist among fermion-antifermion pairs with medium modified masses [Panda, Csörgő, Hama, Krein & SSP, P. L. B512 (2001) 49]



→ **Properties:**

- **Similar positive correlations with unlimited intensity of both fBBC and bBBC**
- **Similar formalism for both bosonic (bBBC) and fermionic (fBBC) Back-to-Back**
- **Expected to appear for $p_T \leq$ 1-2 GeV/c**

Formalism (bosons)

- Infinite medium

$$H_0 = \tfrac{1}{2}\int d\vec{x}\,(\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2) \rightarrow$$
$$\rightarrow \int d^3k\, \omega_k\, a_k^\dagger\, a_k$$

→ Asymptotic (free) Hamiltonian, in the rest frame of matter [operators a and $a^\dagger$]

$$H = H_0 - \tfrac{1}{2}\int d\vec{x}\, d\vec{y}\, \phi(\vec{x})\,\delta M^2(\vec{x}-\vec{y})\,\phi(\vec{y}) \rightarrow$$
$$\rightarrow \int d^3k\, \Omega_k\, b_k^\dagger\, b_k$$

→ In-medium Hamiltonian [operators b and $b^\dagger$]

- Scalar field $\phi(x)$ → quasi-particles propagating with (momentum-dependent) medium-modified effective mass, m_*, related to the vacuum mass, m, by
- Modified mass $$m_*^2\left(\left|\vec{k}\right|\right) = m^2 - \delta M^2\left(\left|\vec{k}\right|\right)$$ $$\left[\delta M^2 = -(\delta m^2 \pm 2m\delta m)\right]$$
- Dispersion relation

$$\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2\left(\left|\vec{k}\right|\right)$$

limit of no-squeezing: $\Omega_k \rightarrow \omega_k$
$\Rightarrow f_k \rightarrow 0 \Rightarrow s_k \rightarrow 0 \wedge c_k \rightarrow 1$

<u>Bogolyubov transformation</u>

$$\begin{cases} a^\dagger{}_k = c_k^*\, b^\dagger{}_k + s_{-k}\, b_{-k} \\ a_k = c_k\, b_k + s_{-k}^*\, b^\dagger{}_{-k} \end{cases}$$

$$f_k = \tfrac{1}{2}\ln(\omega_k / \Omega_k)$$ ↔ <u>Squeezing parameter</u>

$$c_k = \cosh[f_k] \qquad s_k = \sinh[f_k]$$

Full Correlation Function ($\pi^0\pi^0$, $\phi\phi$)

$$\langle a^\dagger_{k_1} a^\dagger_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^\dagger_{k_1} a_{k_1} \rangle \langle a^\dagger_{k_2} a_{k_2} \rangle \pm \langle a^\dagger_{k_1} a_{k_2} \rangle \langle a^\dagger_{k_2} a_{k_1} \rangle$$

NOTATION

$$N_1(\vec{k}_i) = \omega_{k_1} \frac{d^3N}{d^3k} = G_c(\vec{k}_i, \vec{k}_i) \equiv G_c(i,i) = \omega_{k_1} \langle a^\dagger_{k_i} a_{k_i} \rangle$$ → Spectra

$$G_c(\vec{k}_1, \vec{k}_2) \equiv G_c(1,2) = \sqrt{\omega_{k_1}\,\omega_{k_2}}\,\langle a^\dagger_{k_1} a_{k_2} \rangle$$ → Chaotic amplitude

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \frac{\left|G_c(1,2)\right|^2}{G_c(1,1)\,G_c(2,2)}$$

HBT ($K^\pm K^\pm$)

Full Correlation Function ($\pi^0\pi^0$, $\phi\phi$)

$$\langle a^\dagger_{k_1} a^\dagger_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^\dagger_{k_1} a_{k_1} \rangle \langle a^\dagger_{k_2} a_{k_2} \rangle + \langle a^\dagger_{k_1} a_{k_2} \rangle \langle a^\dagger_{k_2} a_{k_1} \rangle + \langle a^\dagger_{k_1} a^\dagger_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

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$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \frac{\left|G_c(1,2)\right|^2}{G_c(1,1)G_c(2,2)} + \frac{\left|G_s(1,2)\right|^2}{G_c(1,1)G_c(2,2)}$$

HBT ($K^\pm K^\pm$)

BBC

K^+K^-

Finite system expanding with non-relat. flow

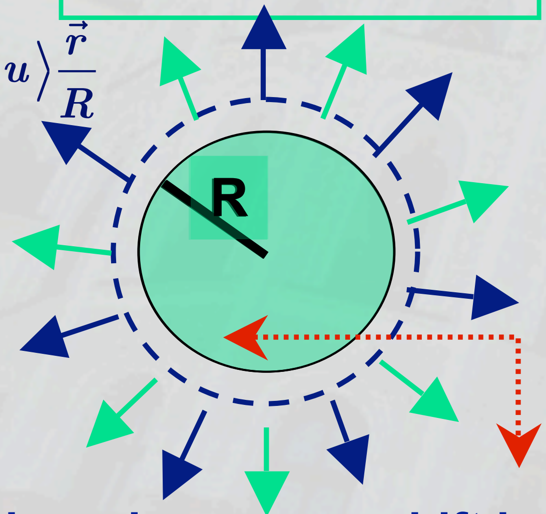
— Non-relativistic flow

— Neglecting flow effects on squeezing parameter $f_{i,j}$

— Simplest finite squeezing vol. profile → analytical calculations: 3-D Gaussian → circular cross-sectional area of radius R

$$\approx \exp[-\vec{r}^2 / (2R^2)]$$

$$\vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$



Region where mass-shift is non-vanishing

Freeze-out

$$\tilde{F}(\Delta\tau) = \int dt\, E_{i,j}\, F(\tau_f)\, e^{-iE_{i,j}(\tau-\tau_0)}\, d\tau_f$$

— Sudden freeze-out [$\delta(\tau-\tau_0)$]

$$\tilde{F}(\Delta\tau) = E_{i,j} e^{-2iE_{i,j}\cdot\tau_0}$$

— Finite emission interval $[\theta(\tau-\tau_0)e^{-(\tau-\tau_0)/\Delta\tau} / \Delta\tau]$

$$\tilde{F}(\Delta\tau) = \frac{E_{i,j}}{[1+(E_{i,j}\Delta\tau)^2]}$$

— Lévy-type distribution (fits of PHENIX correlat.)

$$\tilde{F}(\Delta\tau) = E_{i,j} \exp\{-[\Delta\tau(\omega_1+\omega_2)]^\alpha\}$$

$$n_{i,j}(x) \sim \exp\left[-\left(K^\mu_{i,j} u_\mu - \mu(x)\right) / T(x)\right]$$

Hydro parameterization → $\dfrac{\mu(x)}{T(x)} = \dfrac{\mu_0}{T(x)} - \dfrac{\vec{r}^2}{2R^2}$

$$u^\mu = \gamma(1,\vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

$$\gamma = (1-\vec{v}^2)^{-1/2} \approx 1 + \tfrac{1}{2}\vec{v}^2 \quad [\mathcal{O}(v^2)]$$

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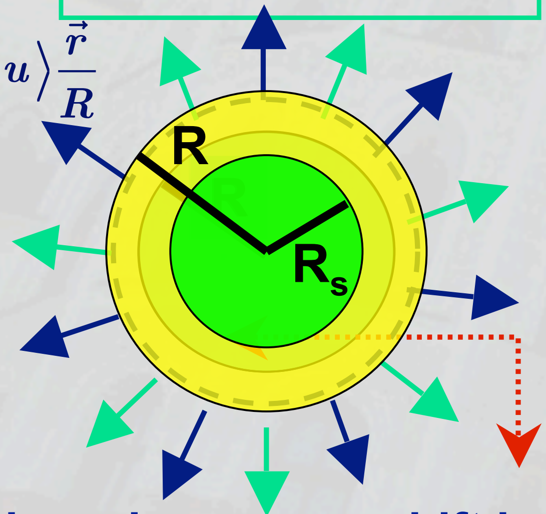

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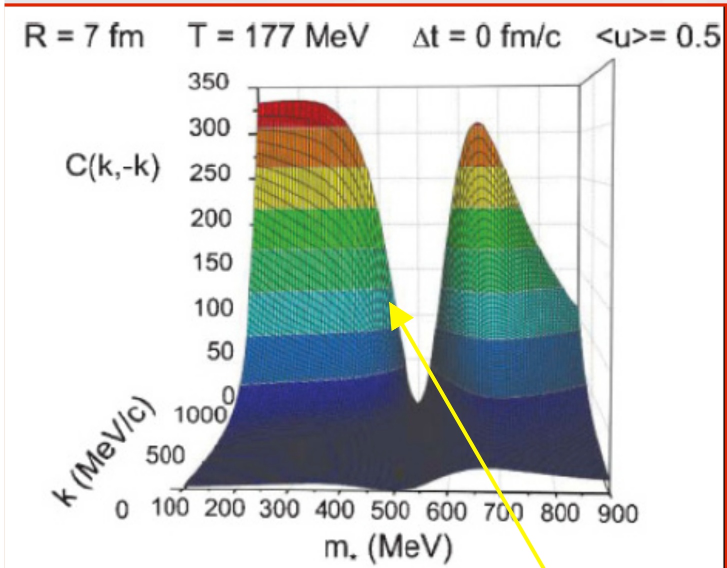
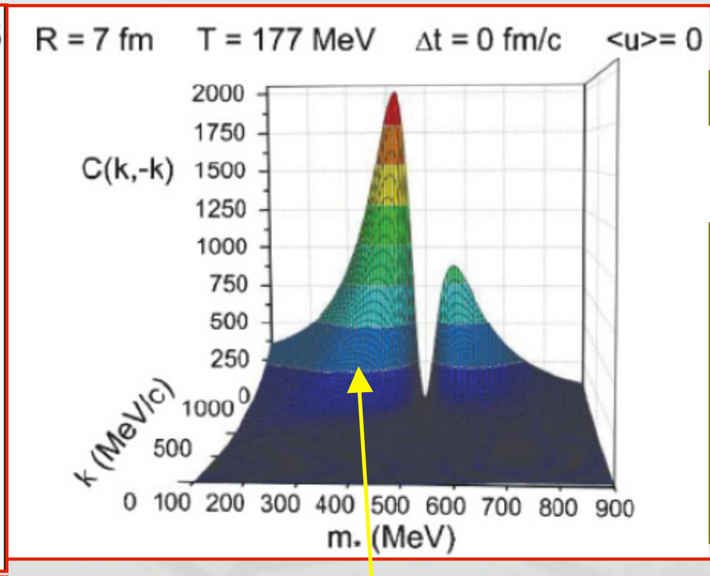
K^+K^- BBC pairs $\rightarrow$ m_* scan

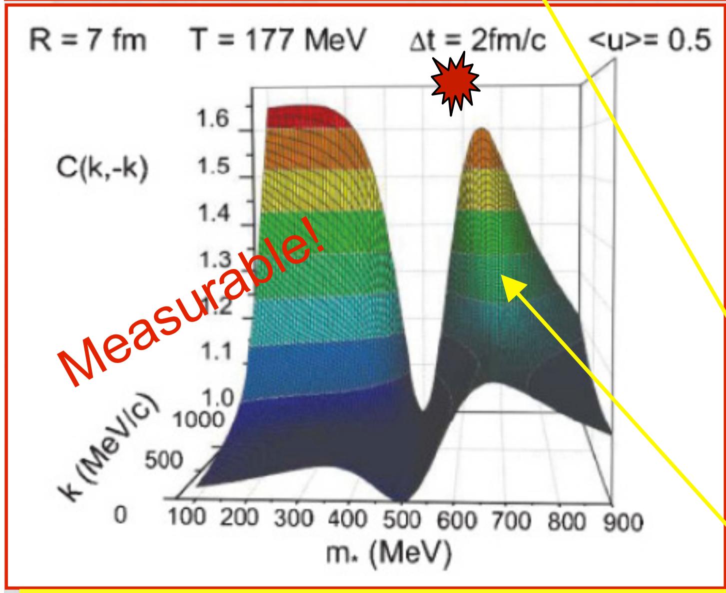


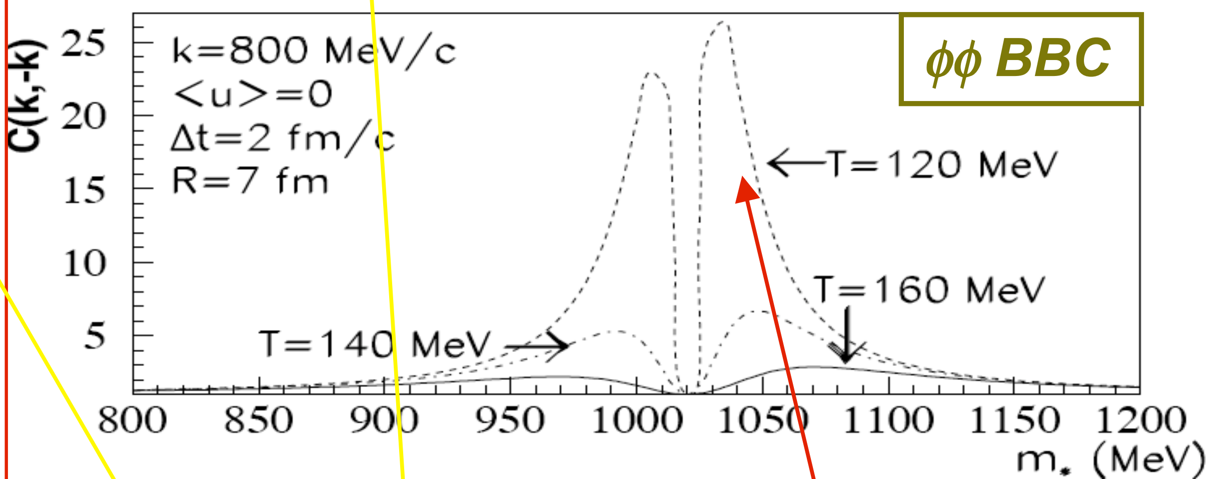


Lorenzian emission
- reduction ~ 1/20

Temperature ↗
- intensity ↘

Flow →increase
(mainly @ low k)



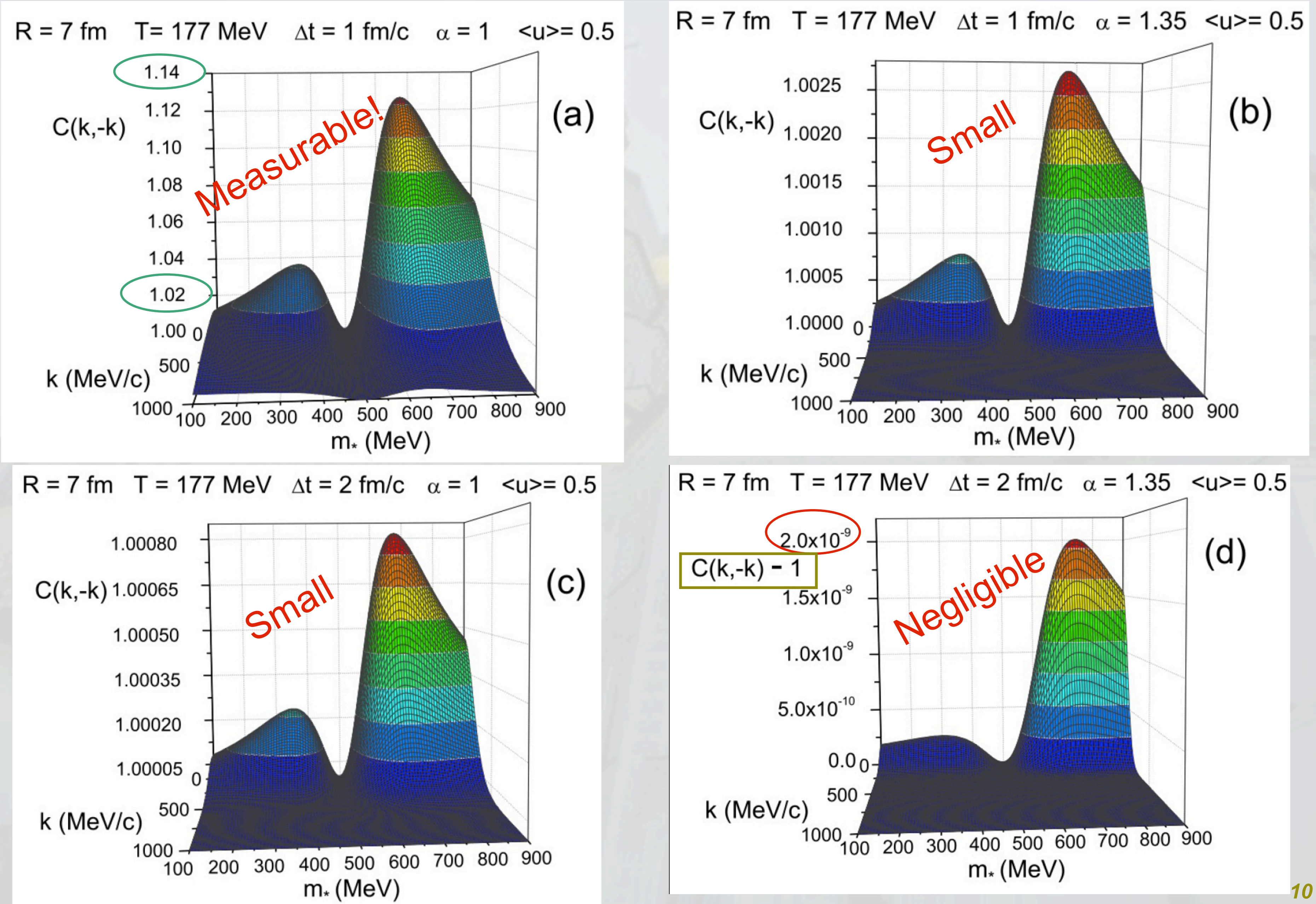


φφ BBC

Similar behavior for <u>=0 & <u>=0.5

Maximal values for each (m_*,|k|), i.e., for strict back-to-back pairs [~ equiv. intercept of HBT correlation function]

Emission: Lévy distribution in time



Role of time emission distribution

- Sudden (instantaneous) freeze-out $\rightarrow$ preserves the full intensity of the squeezed correlation signal
- Lorentzian (Fourier transform of exponential emission) $\rightarrow$ drastic reduction (2 - 3 orders of magnitude smaller)
- Lévy-type distribution:
  - If α=1 $\longrightarrow$ $\Delta\tau$=1fm/c $\rightarrow$ even smaller but measurable
    2 fm/c $\rightarrow$ BBC too small
  - If α=2 $\longrightarrow$ $\Delta\tau$=1fm/c $\rightarrow$ BBC too small
    2fm/c $\rightarrow$ effect is washed out
- But clearly: distribution unknown *a priori* $\rightarrow$ let us look for the effect and then try to parametrize it afterwards
  - From now on $\rightarrow$ Instanteous & Lorentzian emission

(for more details $\longrightarrow$ ArXiv: 1006.5899, PRC in press)

For the experimental search

m_* scan $\Rightarrow$

1) Necessary → maximum values of squeezed corr.
$$C_s(m_*, \vec{k}_1 = -\vec{k}_2 = \vec{k})$$

2) m_* → in-medium, not measurable

3) Momenta are measurable with finite precision
$$|\vec{k}_1| - |\vec{k}_2| \leq \Delta\vec{k} \neq 0$$

Relativistic extension →
(M. Nagy)

$$Q_{back} = (\omega_1 - \omega_2, \vec{k}_1 + \vec{k}_2) = (q_{12}^0, 2\vec{K}_{12})$$

$$Q_{bbc}^2 = -Q_{back}^2 \approx (2\vec{K}_{12})^2$$

Non-relativistic limit

Two main possibilities:

1) Rewrite theoretical $C_s(k_1,k_2)$ in terms of K and q:

$$2 * \vec{K}_{i,j} = (\vec{k}_i + \vec{k}_j)$$
$$\vec{q}_{i,j} = (\vec{k}_i - \vec{k}_j)$$

The effect is **maximal** for

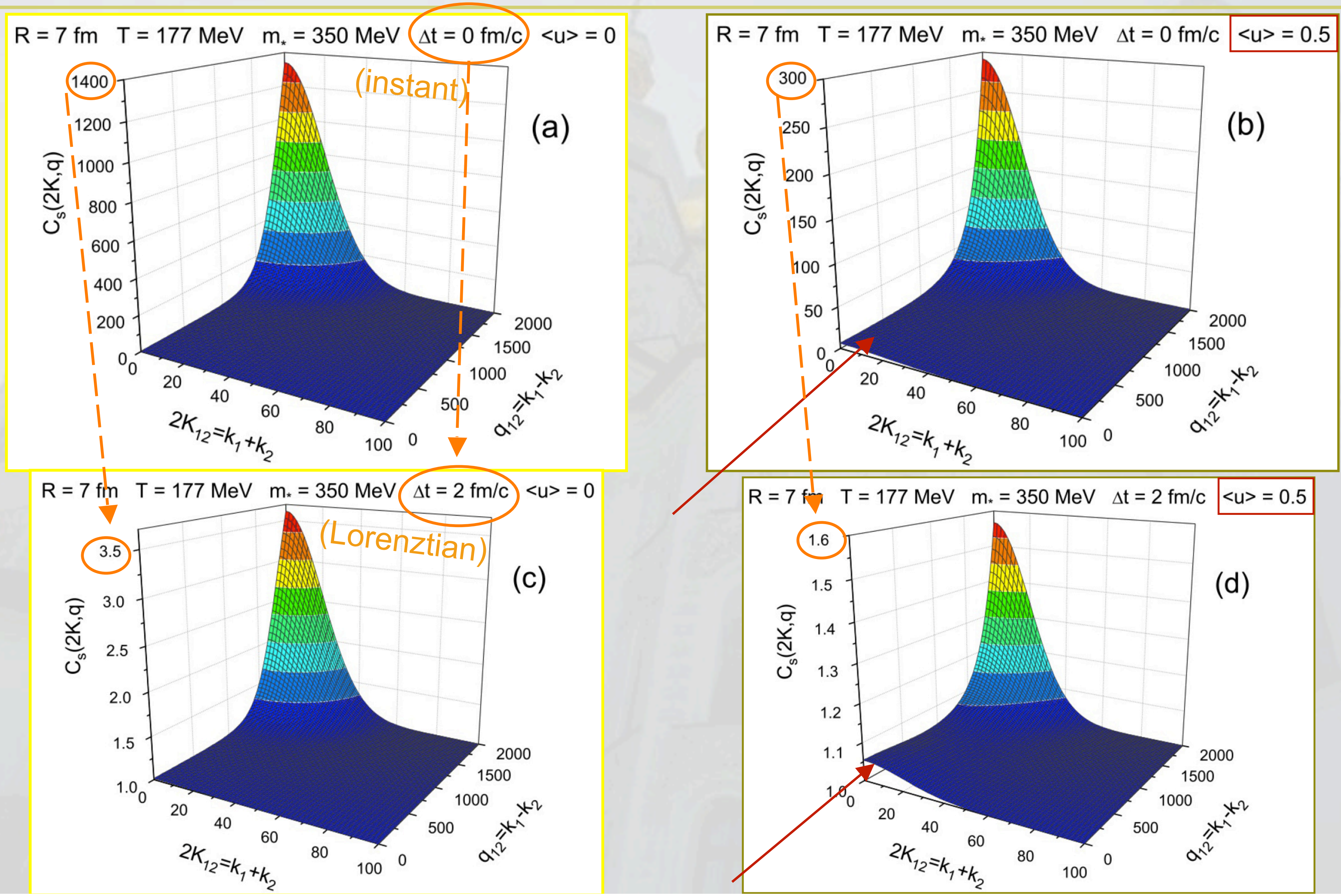
$$\vec{k}_1 = -\vec{k}_2 = \vec{k}$$

i.e., for $\vec{K}_{12} = 0$ → study for different values of q

2) Combine particle-antiparticle pair
→ Theory: generate (k_1,k_2)–simulation
→ Experiment: combine pairs $SEvent/DEvent$ (special way)

Squeezed correlations of K^+K^- pair: predictions

Choose first maximum of $C_s(k_1,k_2)$ in $(m_*,\ \vec{k}_1 = -\vec{k}_2 = \vec{k})$ plane $\leftrightarrow m_*$ = 350MeV



First experimental search for hadronic squeezed states

- **At the WPCF 2009 $\rightarrow$ Martón Nagy showed**
  - **PHENIX Preliminary data $\rightarrow$ shown today (Maté)**

$$C_s(k_1,k_2) \text{ vs. } (|k_1+k_2|,|k_1-k_2|)$$

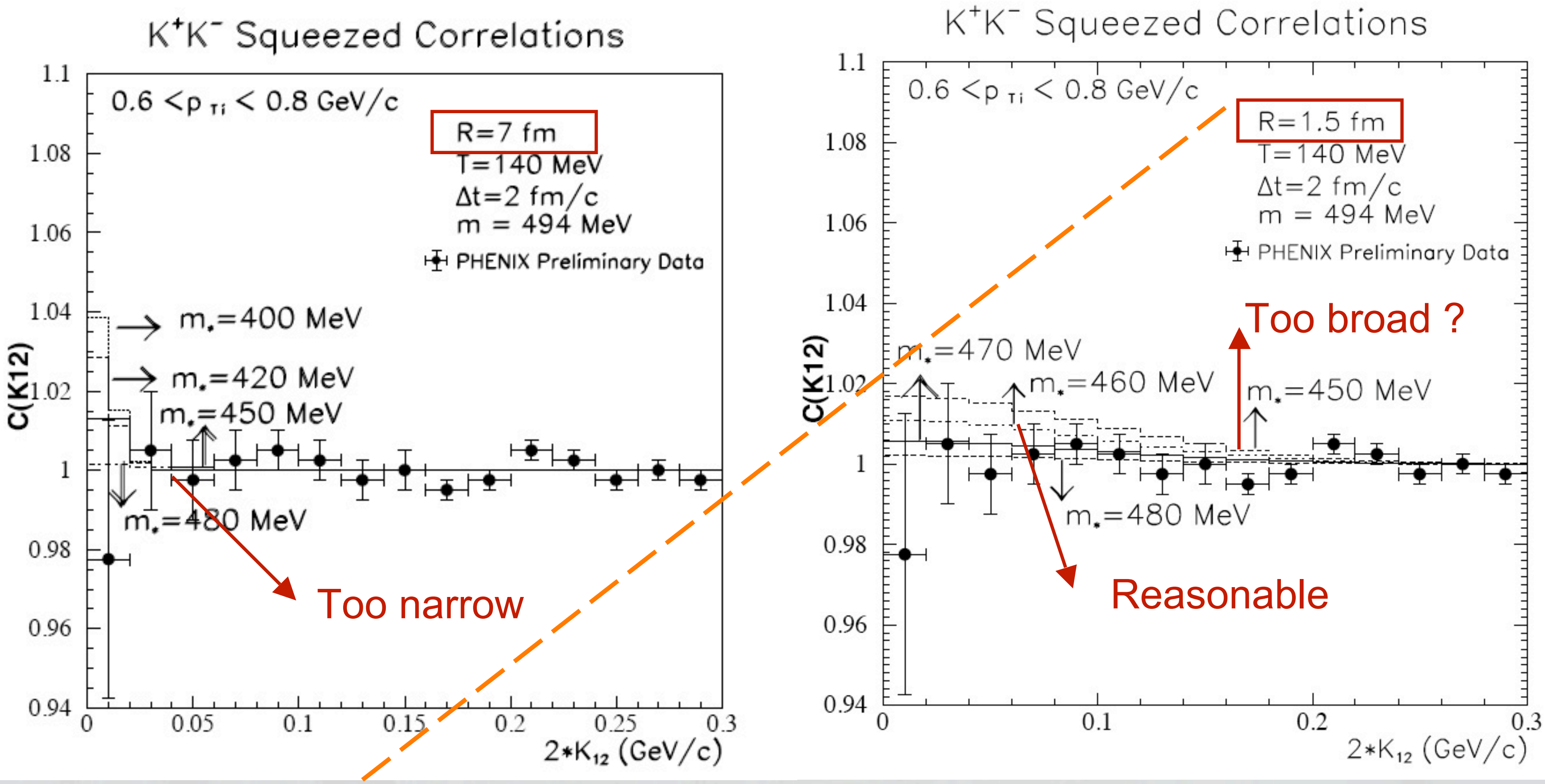
- K^+K^-, $p\bar{p}$ **and** $\pi^+\pi^-$.

  (compatible with our model for bosons)

- **How do the results compare with our model?**

The model & PHENIX preliminary data

- **Squeezing volume = system volume (~ R^3)**



R=1.5fm ?? T=140?? → Phenix data [PRL 103, 142301 (2009)] → R ≈ 3.2 fm; T=177 MeV

The model & PHENIX preliminary data

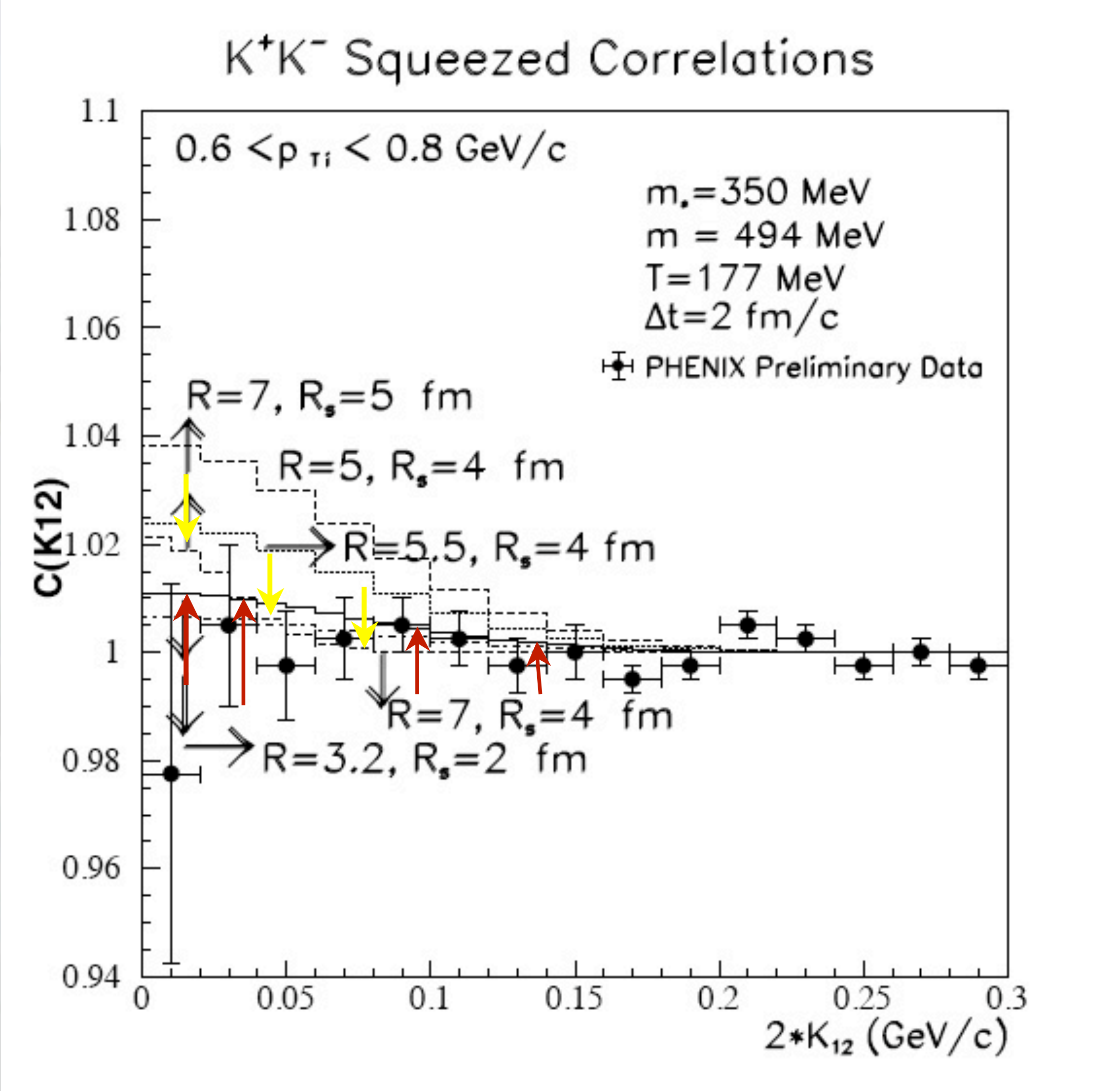
[Model →
PRC 73,044904 (2006)]

Squeezing volume < system volume
R_s → squeezing &
R → system radii

Therefore

- R=7, R_s=5 fm → a possibility
- R=3.2, R_s=2 fm → also possible

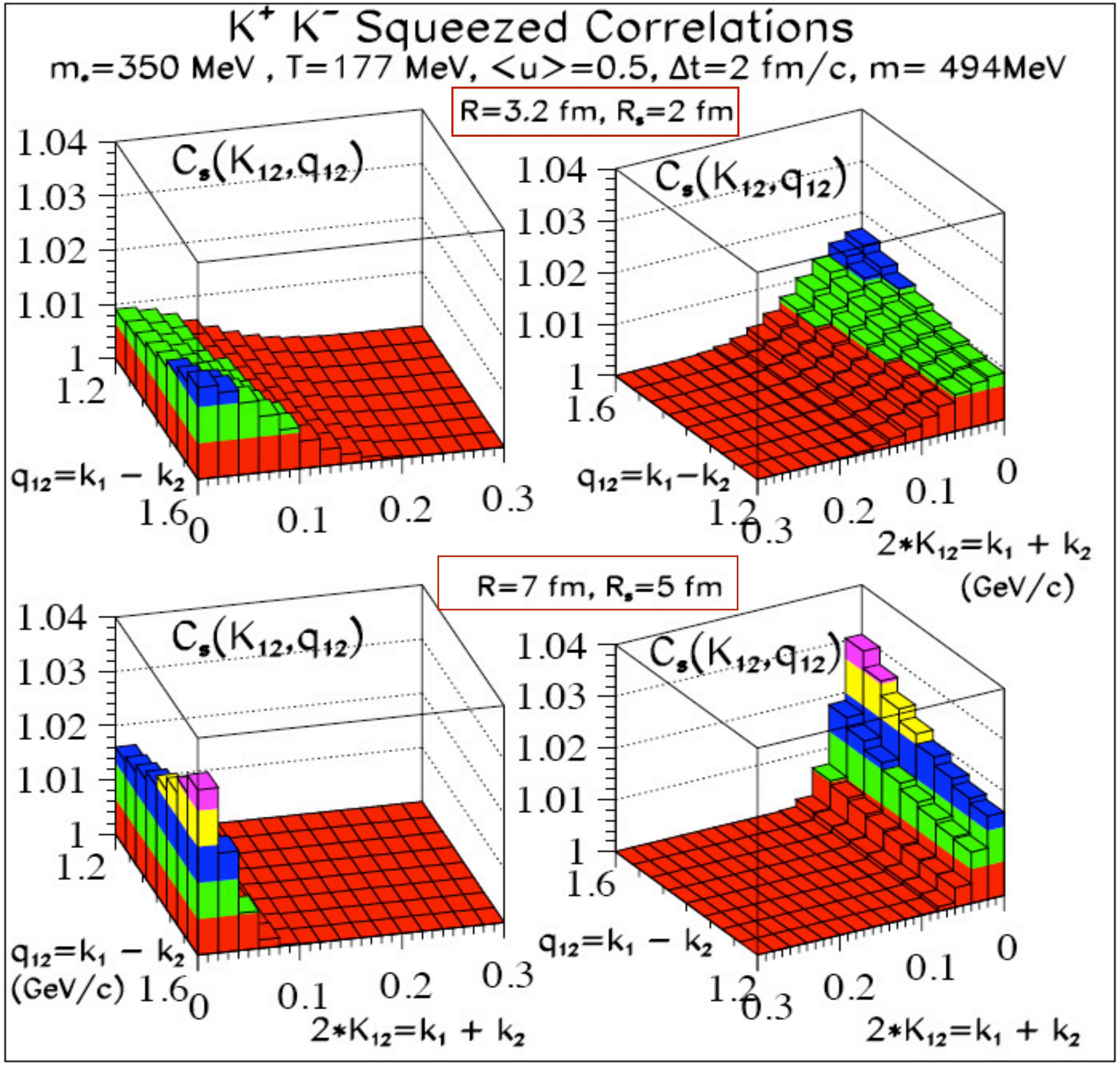
Result → ambiguous but not negative!



Look into $C_s(|k_1+k_2|,|k_1-k_2|)$

- Higher statistics required
- Keeping bining as before

→

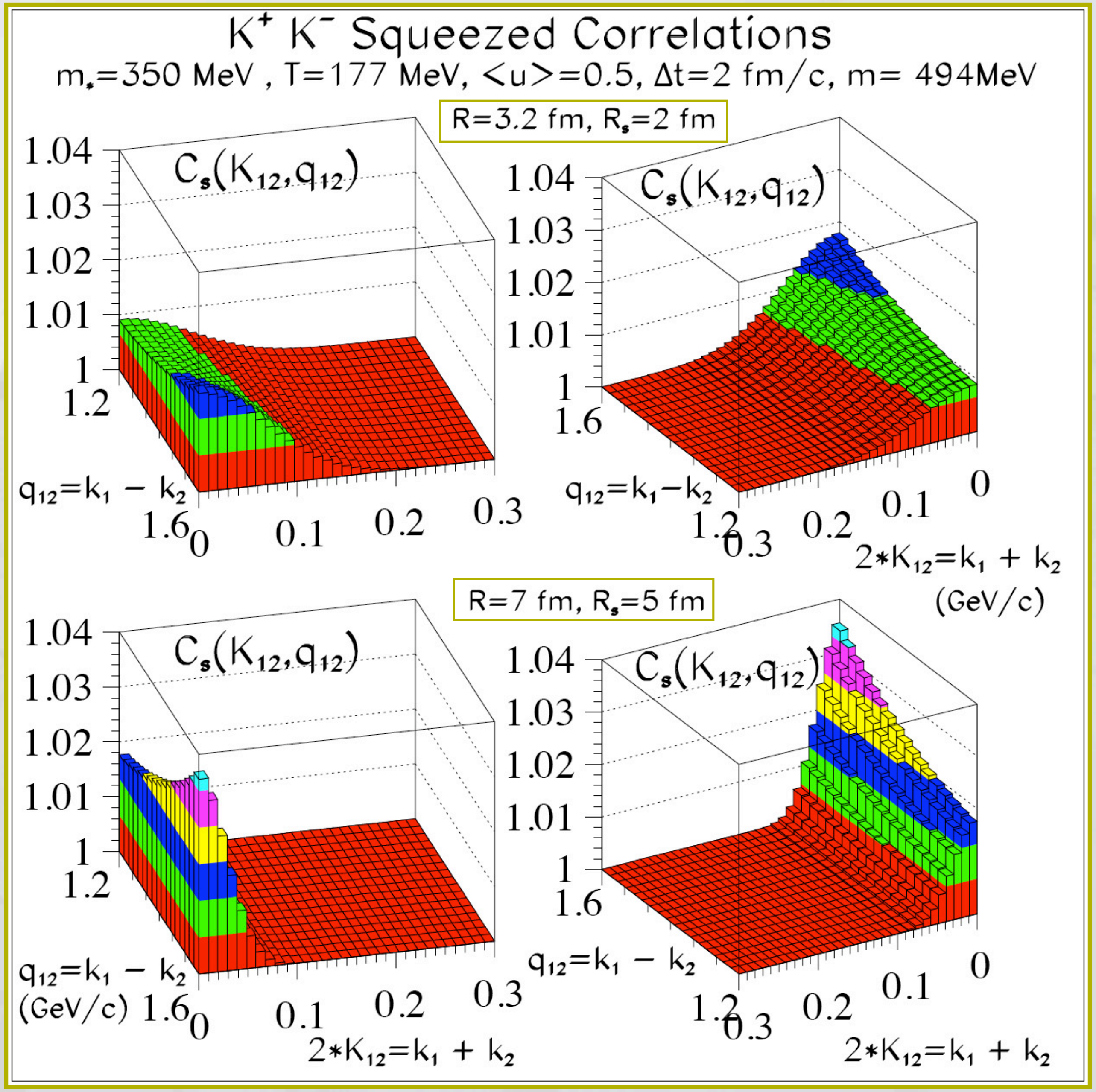


Look into $C_s(|k_1 + k_2|, |k_1 - k_2|)$

- In case statistics allow smaller bins →

→ Clearer distinction

→



Comments & Conclusions

- **1– Considering K^+K^- squeezed correlations**
  - Focused on the role of the time emission interval
    - Lorentzian time factors $\longrightarrow$ strongly suppresses squeezing signal
    - Lévy time factor $\longrightarrow$ reduces the intensity even more drastically:
      - » **α=1**; **$\Delta\tau$=1fm/c** $\rightarrow$ still measurable ; **$\Delta\tau$=2fm/c** $\rightarrow$ too small
      - » **α=2**; **$\Delta\tau$=1fm/c** $\rightarrow$ too small ; **$\Delta\tau$=2fm/c** $\rightarrow$ negligible
  - However, a priori, emission process is unknown
- **2– Supposing nature favors Lorentzian factor**
- **3– Comparison with PHENIX preliminary data**
    - Compatible with squeezing in part of the system
    - **Results still inconclusive but do not exclude the signal so far**
    - $C_s(k_1+k_2, k_1-k_2)$ vs. $(k_1+k_2\,,\,k_1-k_2)$ enhances the differences
    - Higher statistics required $\rightarrow$ perhaps we already have the signal
    - If not yet at RHIC energies, perhaps at LHC soon!

EXTRA SLIDES

Correlation for strict BBC pairs

$$C_s(k,-k) \sim 1 + \frac{|c_0|^2 |s_0|^2 R^6}{|s_0|^4 R^6} \sim 1 + \frac{|c_0|^2}{|s_0|^2} \xrightarrow[\substack{s_0 \to 0 \\ c_0 \to 1}]{m \sim m_*} \text{div.}$$

$$2 * K^{\mu}_{i,j} = (k_i + k_j)$$

$$q^{\mu}_{i,j} = (k_i - k_j)$$

$\langle u \rangle = 0\ ;\ |k| \gg 1$

$$k_2 = -k_1 = k$$

$$C_s(k,-k) = 1 + \left\{ |c_0||s_0| \left[R^3 + 2\left(\frac{R^2}{(1 + \frac{m^2 \langle u \rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} \right) \right] \right\}^2 \times$$

$$\left\{ |s_0|^2 R^3 + \left(|c_0|^2 + |s_0|^2 \right) \left(\frac{R^2}{(1 + \frac{m^2 \langle u \rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} + \frac{m^2 \langle u \rangle^2 k^2 / m_*^2}{(1 + \frac{m^2 \langle u \rangle^2}{m_* T}) 2T^2} \right) \right\}^{-2}$$

Squeezed Correlation vs. k_1 & k_2

$2*\vec{K}=\vec{k}_1+\vec{k}_2$ $\vec{q}=\vec{k}_1-\vec{k}_2$

$$G_s(k_1,k_2)=\frac{E_{1,2}}{(2\pi)^{3/2}}c_{12}s_{12}\left\{\begin{array}{l} R^3\exp\left(-\frac{R^2(k_1+k_2)^2}{2}\right)+2\,n_0^*R_*^3\exp\left(-\frac{(k_1-k_2)^2}{8m_*T}\right)\times \\ \exp\left[\left(-\frac{im\langle u\rangle R}{2m_*T_*}-\frac{1}{8m_*T_*}-\frac{R_*^2}{2}\right)(k_1+k_2)^2\right]\end{array}\right\}$$

$2*\vec{K}=\vec{k}_1+\vec{k}_2$

Remember: $2*\vec{K}=\vec{k}_1+\vec{k}_2$, $\vec{q}=\vec{k}_1-\vec{k}_2$

$$G_c(k_i)=\frac{E_{i,i}}{(2\pi)^{3/2}}\left\{\left|s_{ii}\right|^2R^3+n_0^*\;R_*^3\left(\left|c_{ii}\right|^2+\left|s_{ii}\right|^2\right)\exp\left(-\frac{k_i^2}{2m_*T_*}\right)\right\}$$

$$R_*=R\sqrt{\frac{T}{T_*}}$$

$$C_s(\vec{k}_1,\vec{k}_2)=1+\frac{\left|G_s\left(\vec{k}_1,\vec{k}_2\right)\right|^2}{G_c\left(\vec{k}_1,\vec{k}_1\right)G_c\left(\vec{k}_2,\vec{k}_2\right)}$$

$$T_*=(T+\frac{m^2}{m_*}\langle u\rangle^2)$$

Formalism (fermions)

$$H = H_0 + H_I \quad ; \quad H_0 = \int d\vec{x} : \bar{\psi}(x)(-i\vec{\gamma}.\vec{\nabla} + M)\psi(x):$$

$$\psi(x) = \frac{1}{V}\sum_{\lambda,\lambda',\vec{k}} (u_{\lambda,\vec{k}} a_{\lambda,\vec{k}} + v_{\lambda',-\vec{k}} a^{\dagger}_{\lambda',-\vec{k}}) e^{i\vec{k}.\vec{x}}$$

$$\langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^{\dagger}_{k_1} a_{k_1} \rangle \langle a^{\dagger}_{k_2} a_{k_2} \rangle - \langle a^{\dagger}_{k_1} a_{k_2} \rangle \langle a^{\dagger}_{k_2} a_{k_1} \rangle + \langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

- System described by quasi-particles → medium effects taken into account through self-energy function
- For spin-1/2 particles under mean fields in a many body system:

$$\sum{}^{s} + \gamma^0 \sum{}^{0} + \gamma^i \sum{}^{i}$$ → to be determined by detailed calculation

- Σ^s → notation: $\Sigma^s(k) = \Delta M(k)$
- Σ^1 → very small → neglected
- Σ^0 → weakly-dependent on momentum→totally thermalized medium: $\mu_* = \mu - \Sigma^0$ → (results for net barion number)
- Hamiltoniana H_1 → describes a system of quasi-particles with mass-dependent momentum $m_* = m - \Delta M(k)$

bBBC & fBBC - formalism summary

- **Bosonic BBC**

$$c_k = \cosh[f_k] \; ; \; s_k = \sinh[f_k]$$

$$\begin{cases} a^\dagger_k = c_k b^\dagger_k + s_{-k} b_{-k} \\ a_k = c_k b_k + s^*_{-k} b^\dagger_{-k_1} \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \frac{1}{2}\log\left(\frac{\omega_k}{\Omega_k}\right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

- **Fermionic BBC**

$$c_k = \cos[f_k] \; ; \; s_k = \sin[f_k]$$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}^\dagger_{\lambda',-k} \end{pmatrix} = \begin{pmatrix} c_k & \frac{f_k}{|f_k|} s_k A \\ -\frac{f_k^*}{|f_k|} s_k^* A^\dagger & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}^\dagger_{\lambda',-k} \end{pmatrix}$$

$$A = [\chi^\dagger_\lambda (\sigma.\hat{k}) \tilde{\chi}_{\lambda'}] \; ; \; A^\dagger = [\tilde{\chi}^\dagger_{\lambda'} (\sigma.\hat{k})^\dagger \chi_\lambda]$$

$$\begin{cases} \tilde{\chi}_{\lambda'} = -i\sigma^2 \chi_{\lambda'} \; ; \; \hat{k} = \vec{k}/|\vec{k}| \\ \text{→ is a Pauli spinor} \\ \tan(2f_k) = -\frac{|k|\Delta M(k)}{\omega_k^2 - \Delta M(k) M} \\ m_*(k) = m - \Delta M(k) \\ \omega_k^2 = m^2 + \vec{k}^2 \; ; \; \Omega_k^2 = m_*^2 + \vec{k}^2 \end{cases}$$