Highlights on femtoscopy in e^+e^- and hp collisions

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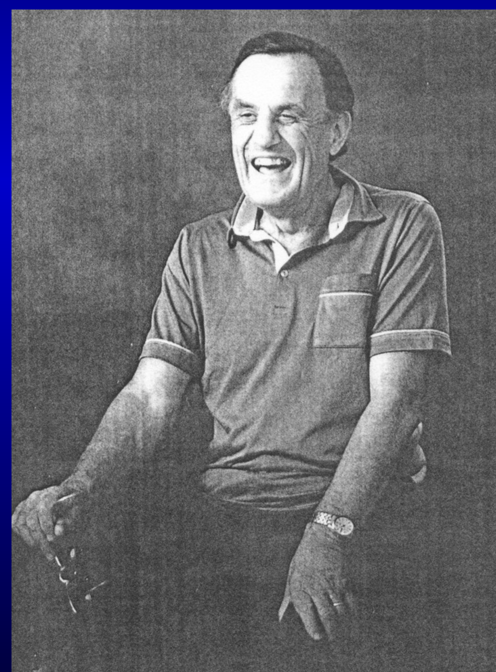
HBT story, intensity interferometry
GGLP, Bose-Einstein Correlations
Femtoscopy
Applications:
Multiplicity dependence
Shape analysis, femtoscopic movie
Other femtoscopic correlations

An old/new look at the stars

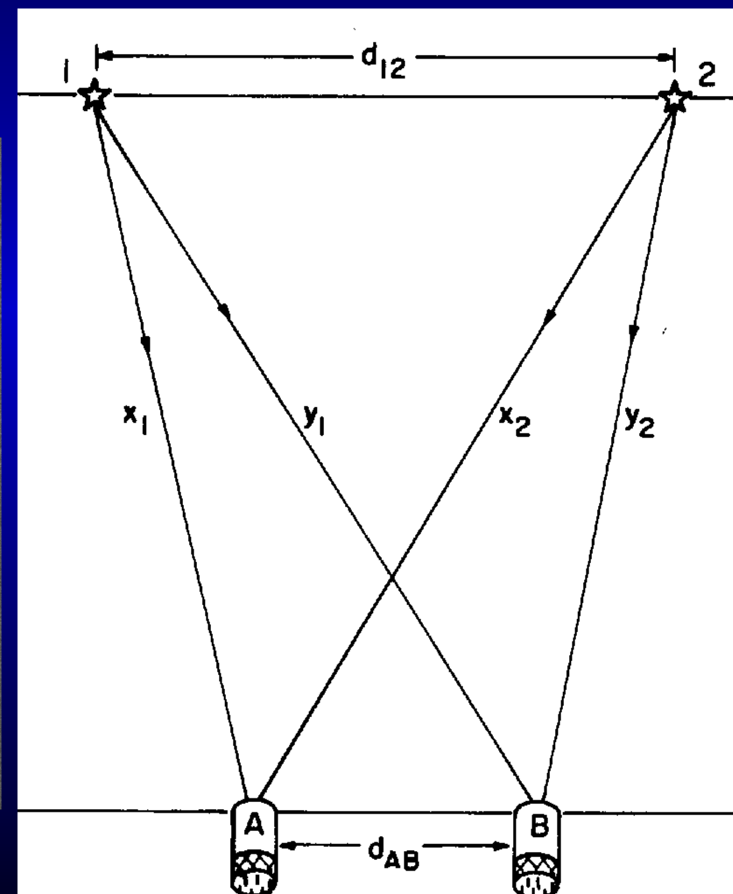
Intensity interferometry in radio astronomy

Angular diameter of a main sequence stars

Figure 10.1 The first stellar intensity interferometer; the pilot model of the stellar intensity interferometer at Jodrell Bank in 1955. Two Army searchlights were used to make the first measurement of the angular diameter of a main sequence star (Sirius).



R.Hanbury Brown



R. Hanbury Brown and R. Q. Twiss

Engineers, worked in radio astronomy

Two people: Robert Hanbury Brown and Richard Q. Twiss

- Robert, Hanbury and Richard: all given names…

„Interference between two different photons can never occur."
P. A. M. Dirac, The Principles of Quantum Mechanics, Oxford, 1930

„In fact to a surprising number of people the idea that the arrival of photons at two separated detectors can ever be correlated was not only heretical but patently absurd, and they told us so in no uncertain terms, in person, by letter, in print, and by publishing the results of laboratory experiments, which claimed to show that we were wrong …"

"I was a long way from being able to calculate, whether it would be sensitive enough to measure a star. To do that one has to be familiar with photons and as an engineer my education in physics had stopped far short of the quantum theory. Perhaps just as well, otherwise like most physicists I would have come to the conclusion that the thing would not work – ignorance is sometimes a bliss in science"

Bose-Einstein or HBT correlations

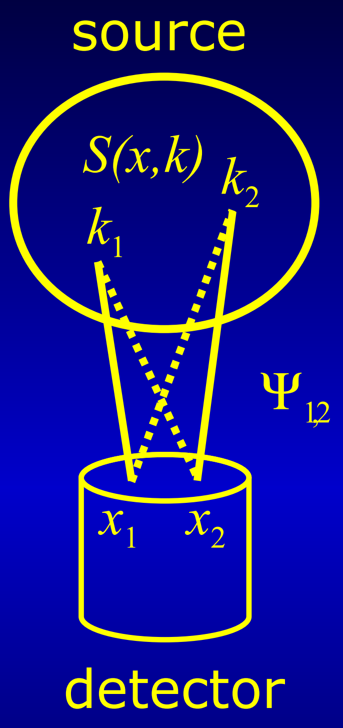
Two plane-waves:

Bosons:

symmetrization

$$\Psi_1 = \mathrm{e}^{-ik_1x_1}$$

$$\Psi_2 = \mathrm{e}^{-ik_2x_2}$$



$$\Psi_{1,2} = \frac{1}{\sqrt{2}}(\mathrm{e}^{-ik_1x_1}\mathrm{e}^{-ik_2x_2} + \mathrm{e}^{-ik_1x_2}\mathrm{e}^{-ik_2x_1})$$

$$N_1(k_1) = \int S(x_1,k_1)|\Psi_1|^2 \mathrm{d}x_1$$

S(x,k): source distribution.

Picture: for HBT, formulas for femtoscopy x ↔ k

Two-particle spectrum (momentum-distribution):

$$N_2(k_1,k_2) = \int S(x_1,k_1)S(x_2,k_2)|\Psi_{1,2}|^2 \mathrm{d}x_1 \mathrm{d}x_2$$

Approximations: Plane-wave, no multiparticle symmetrization, thermalization …

GFGHKP → GGLP

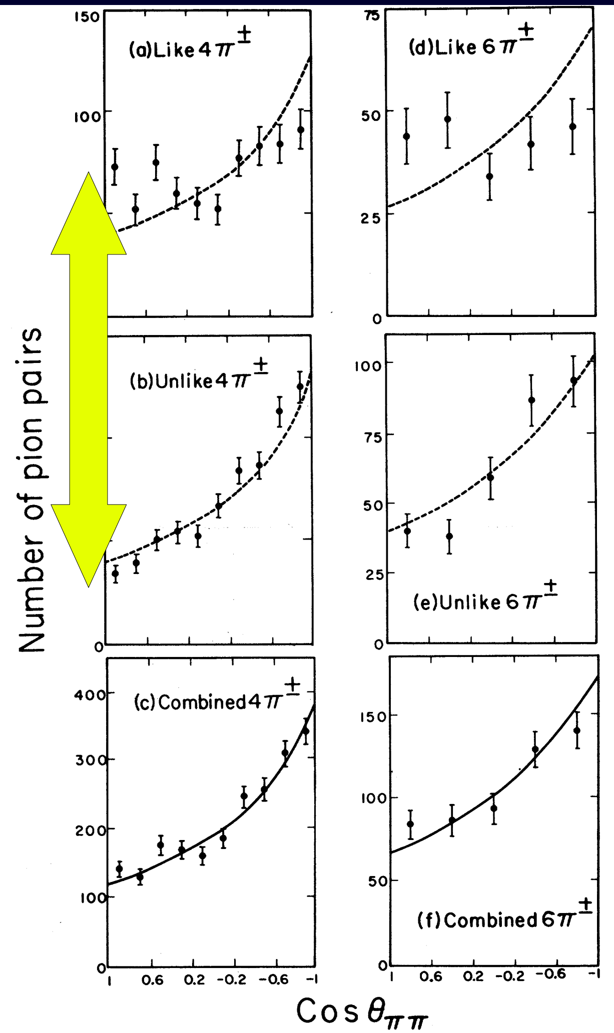


FIG. 1. Distribution of angles between pion pairs as a function of $\cos\theta_{12}$. The curves correspond to calculations on the Lorentz-invariant phase-space (LIPS) model. The deviations of the experimental distribution from the LIPS model are discussed in the text.

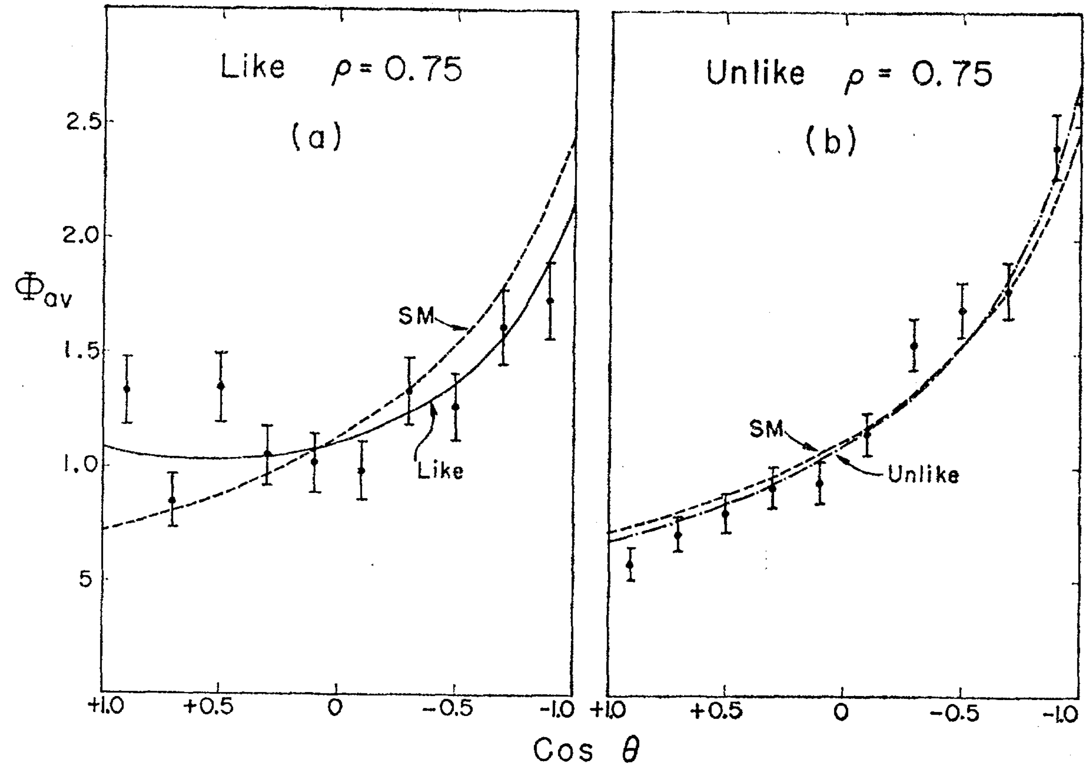


FIG. 6. The functions $\Phi_{av}(\cos\theta)$ computed at $\rho=0.75$ are compared with the experimental distribution of angles between pion pairs. Figures 6(a) and 6(b) give the distributions for like and unlike pions respectively. Also shown in each is the curve for $\Phi_{av}^{SM}(\cos\theta)$, the statistical distribution, without the effect of correlation functions. Here Φ_{av} represents an average of Φ_4, Φ_5, and Φ_6, weighted according to the individual charge channels. The experimental data comes from reference 1 (see also Table I, footnote a).

Kopylov, Podgoretskii, Lednicky

G. Goldhaber, S. Goldhaber, W-Y. Lee and A. Pais (GGLP) :

- explain a HBT like effect in $p+\bar{p}$ reactions at

Kopylov, Podgoretskii, Dubna school

- Start to use correlations as a tool to measure sizes
  - G.I. Kopylov: Like particle correlations as a tool...
    - Phys. Lett. B 50, 474 (1974)
- Interference of particles emitted by moving sources
  - G.I. Kopylov, M. I. Podgoretsky
    - Yad.Fiz.18:656-666 (1973)
    - Yano, Koonin, Podgoretsky (YKP) parametrization

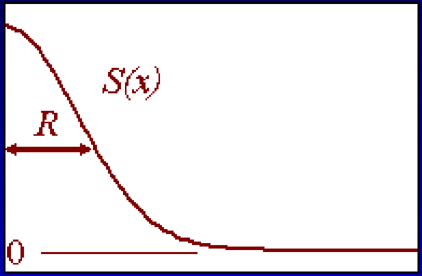
- Non-identical particle interferometry: effects of fsi
  - Sequence of particle emission in principle can be obtained
  - R. Lednicky, Ljuboshitz
    - Yad.Fiz.35:1316-1330,1981

Lednicky: Coined the name of Femtoscopy (nucl-th/0112011)

Again, what brings us all this?

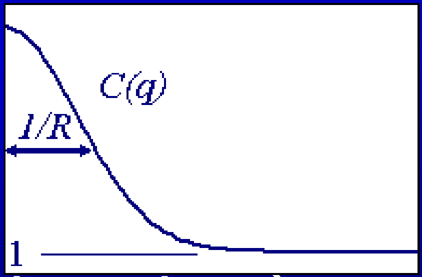
If the source is approximated with Gaussian:

$$S(x) \sim \exp\left(-\frac{r_x^2}{2R_x^2} - \frac{r_y^2}{2R_y^2} - \frac{r_z^2}{2R_z^2}\right)$$



Then the correlation function is also Gaussian:

$$C(q) - 1 \sim \exp\left(-q_x^2 R_x^2 - q_y^2 R_y^2 - q_z^2 R_z^2\right)$$



- **These are the so-called HBT radii**

If transformed to the out-side-long system (not invariant)

Out: direction of the mean transverse momentum of the pair

Side: orthogonal to out

Long: beam direction

$$C(q) = 1 + \lambda \exp\left(-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2\right)$$

Not necessarily reflecting the geometrical size

- Take a hydro model of an expanding ellipsoid…

Experiments: UA1, NA22, L3, OPAL…

GFGHKP: 30 in. Bubble chamber experiment, $\bar{p}+p$, $\sqrt{s}_{NN}$ = 2.1 GeV, LRL,

2500 events, 2106+532 = 2638 total number of pairs

EHS/NA22: Bubble chamber experiment at CERN SPS $\sqrt{s}_{NN}$ = 22 GeV, SPS

– 25 k π^+p and 29k K^+p events

UA1: $p+\bar{p}$ experiment at CERN SppS sqrt(s) = 630 GeV

– p_T > 0.15 GeV/c, $|\eta| < 3$, 45° < $|\phi|$< 135°

– 1.2 x 10^6 NSD events, $|\Delta k|$ ~ 8 MeV

L3, OPAL, ALEPH, DELPHI:

$e^+ e^-$ annihilations at LEP. 2 jets and 3 + jets, $\sqrt{s}_{NN}$ = 91.2 GeV

~ 10^6 events (hadronic Z^0 decays) + …

UA1: Non-Gaussian distributions

Correlations do NOT have to be Gaussian

Non-Gaussian tails in 630 GeV p+p

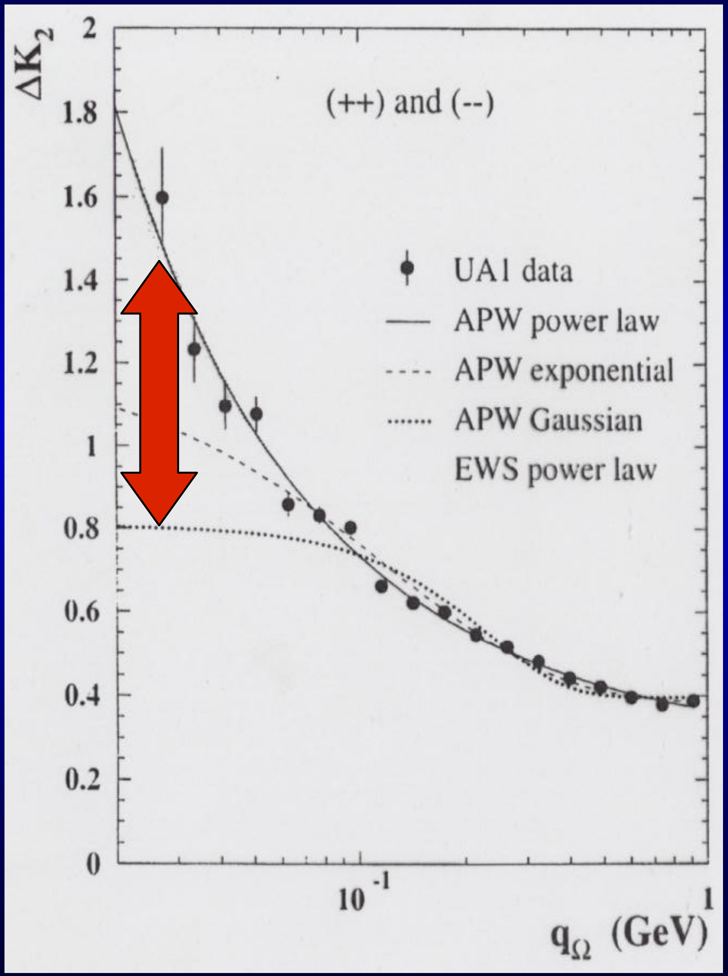
Log scale in q, many low q bins

Partial coherence: 1 + 2 terms

Best Gaussians/exponentials FAIL

$$\text{Gaussian:} \quad d_{ij} = \exp(-r^2 q_{ij}^2)\,,$$
$$\text{exponential:} \quad d_{ij} = \exp(-r q_{ij})\,,$$
$$\text{power law:} \quad d_{ij} = q_{ij}^{-\alpha}\,.$$

$$k_2^{\rm th} \equiv \frac{C_2}{\rho_1 \otimes \rho_1} = 2\lambda(1-\lambda)d_{12} + \lambda^2 d_{12}^2\,,$$



Eggers, Lipa, Buschbeck, hep-ph/9702235

APW: Andreev, Plümer, Weiner, Int. J. Mod. Phys. A8 4577 (1993).

Gaussian assumption →

meaningless results (CL<0.1 %)

How to check, if the correlation function is really Gaussian ?

UA1: Partial coherence fails

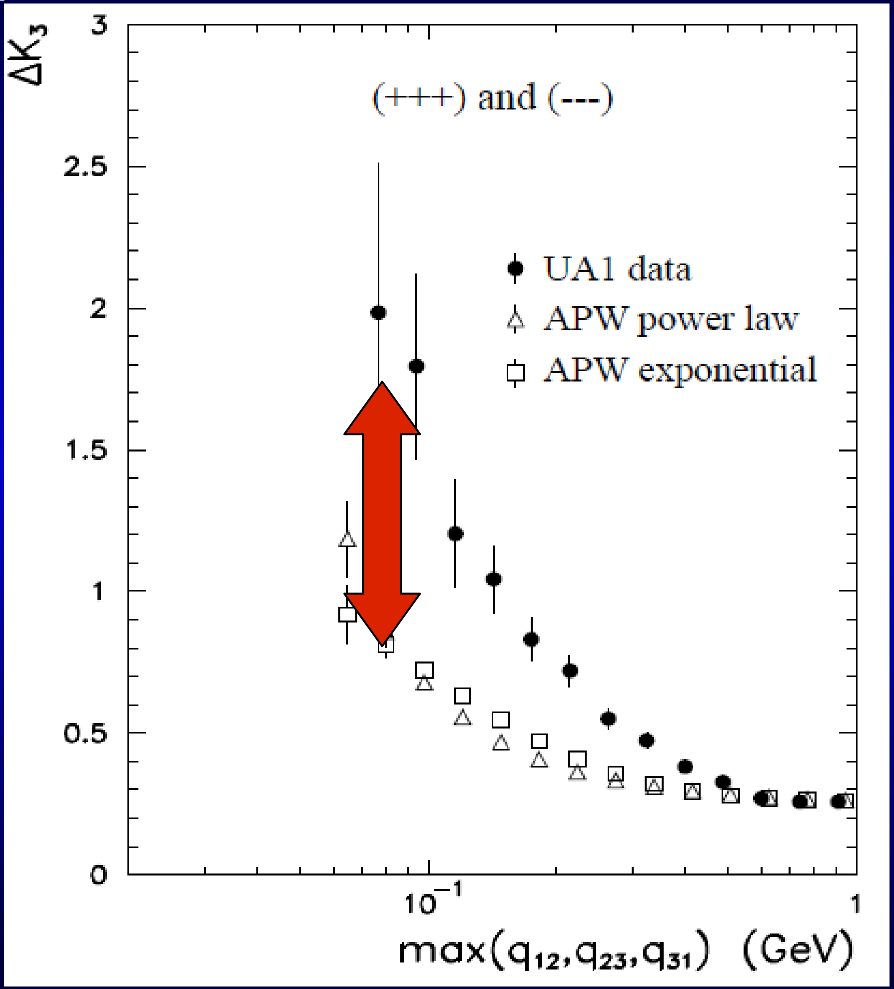
Correlations are NOT due to partial coherence alone

2[rd] and 3[rd] order correlations in 630 GeV p+p NSD events

$$\begin{aligned} \text{Gaussian:} \quad & d_{ij} = \exp(-r^2 q_{ij}^2)\,, \\ \text{exponential:} \quad & d_{ij} = \exp(-r q_{ij})\,, \\ \text{power law:} \quad & d_{ij} = q_{ij}^{-\alpha}\,. \end{aligned}$$

$$k_2^{\rm th} \equiv \frac{C_2}{\rho_1 \otimes \rho_1} = 2\lambda(1-\lambda) d_{12} + \lambda^2 d_{12}^2\,,$$

$$k_3^{\rm th} \equiv \frac{C_3}{\rho_1 \otimes \rho_1 \otimes \rho_1} = 2\lambda^2(1-\lambda)[d_{12}d_{23} + d_{23}d_{31} + d_{31}d_{12}] + 2\lambda^3 d_{12}d_{23}d_{31}\,,$$



3[rd] order correlation: stronger, than from 2[rd] order + partial coh.

→ **How to check, if the source has some partial coherence or not?**

Eggers, Lipa, Buschbeck, hep-ph/9702235

APW: Andreev, Plümer, Weiner, Int. J. Mod. Phys. A8 4577 (1993).

Model independent shape analysis

Advantage and/or disadvantage:

- **Analyse, quantify correlations model independently**

Only two assumptions:

- **The correlations are centered around some point (Q = 0)**

They are short-range type

- **Long range correlations can be removed or measured independently**

Expansion methods to test:

- **Is it Gaussian ? → Edgeworth expansions**
- **Is it Exponential ? → Laguerre expansions**
  - **Based on complete orthog. set of functions**
  - **T. Cs. and S. Hegyi, hep-ph/9912220**
    - » **Not 1+ pos definite**
    - » **Not connected to a source model**

General idea of Expansion

Applied in e+e-, h+p, and in heavy ion reactions:

$$\int dt w(t) h_n(t) h_m(t) = \delta_{n,m},$$

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)},$$

$$f(t) = \sum_{n=0}^{\infty} f_n\, h_n(t),$$

$$f_n = \int dt w(t)\, f(t)\, h_n(t).$$

$$R_2(\mathbf{k}_1, \mathbf{k}_2) = C_2(\mathbf{k}_1, \mathbf{k}_2) - 1.$$

$t = R_I Q_I$ or $t = (R_L Q_L, R_T Q_T)$ or $t = (R_L Q_L, R_{side} Q_{side}, R_{out} Q_{out})$

Let us assume, that the function $g(t) = R_2(t)/w(t)$ is also an element of the Hilbert space H. This is possible, if

$$\int dt\, w(t) g^2(t) = \int dt\, \left[R_2^2(t)/w(t)\right] < \infty, \qquad (6)$$

Is it Gaussian? → Edgeworth Expansion

Model independent, in e+e-, h+p, and in heavy ion reactions:

$$t = \sqrt{2} Q R_E,$$
$$w(t) = \exp(-t^2/2),$$
$$\int_{-\infty}^{\infty} dt\, \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_1(t) = t,$$
$$H_2(t) = t^2 - 1,$$
$$H_3(t) = t^3 - 3t,$$
$$H_4(t) = t^4 - 6t^2 + 3, \ \ldots$$

$$C_2(Q) = \mathcal{N} \Big\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \Big[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2} Q R_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2} Q R_E) + \ldots \Big] \Big\}.$$

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt} \right)^n \exp(-t^2/2).$$

T. Cs., S. Hegyi, hep-ph/9912220

Exponential? → Laguerre expansion

Model independent, in e+e-, h+p, and in heavy ion reactions:

$$t = QR_L,$$
$$w(t) = \exp(-t),$$
$$\int_0^\infty dt\, \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$
$$L_2(t) = t^2 - 4t + 2, \dots .$$

$$C_2(Q) = \mathcal{N}\left\{1 + \lambda_L \exp(-QR_L)\left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!} L_2(QR_L) + \dots\right]\right\}$$

$$L_n(t) = \exp(t)\frac{d^n}{dt^n}(-t)^n \exp(-t).$$

T. Cs., S. Hegyi, hep-ph/9912220

UA1, NA22: more peaked than exponential

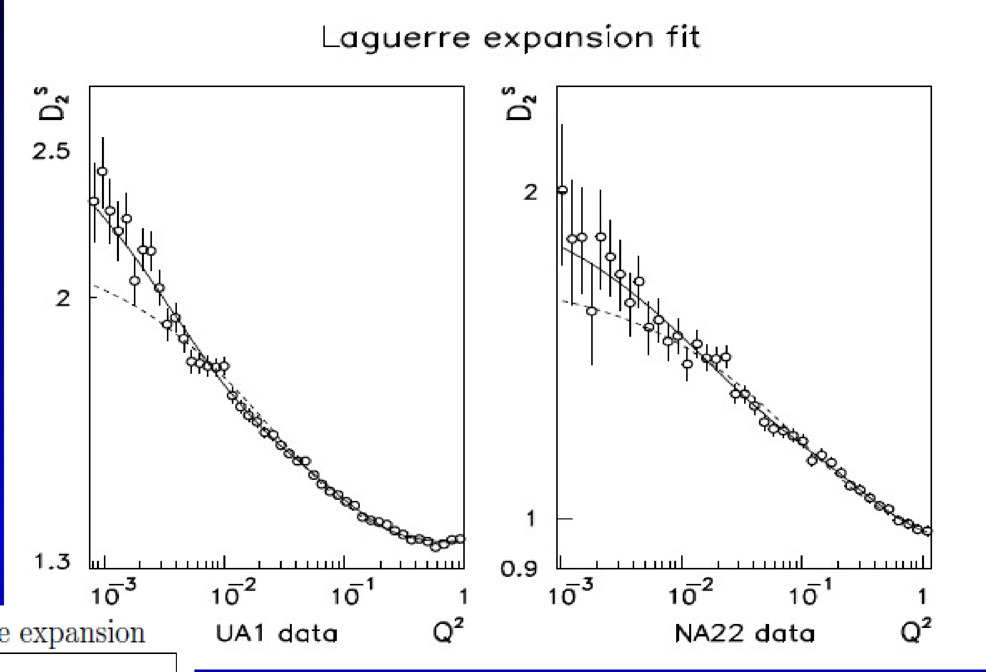
D_2^s: Correlation function

Significantly sharper than best exponential fit: dashed

Note the log scale when binning in Q^2

c_1 and c_2 differ from 0 Significantly

→ Non-exponential shape



Best fits to UA1 and NA22 two-particle correlations using a Laguerre expansion

Parameter	UA1 Value	UA1 Error	NA22 Value	NA22 Error
$\mathcal{N}$	1.355	± 0.003	0.95	± 0.01
λ_L	1.23	± 0.07	1.37	± 0.10
R_L [fm]	2.44	± 0.12	1.35	± 0.14
c_1	0.52	± 0.03	0.63	± 0.06
c_2	0.45	± 0.04	0.44	± 0.06
χ^2/NDF	$41.2/41 = 1.01$		$20.0/34 = 0.59$	

Multivariate generalizations, recent results: see talk M. de Kock/Saturday

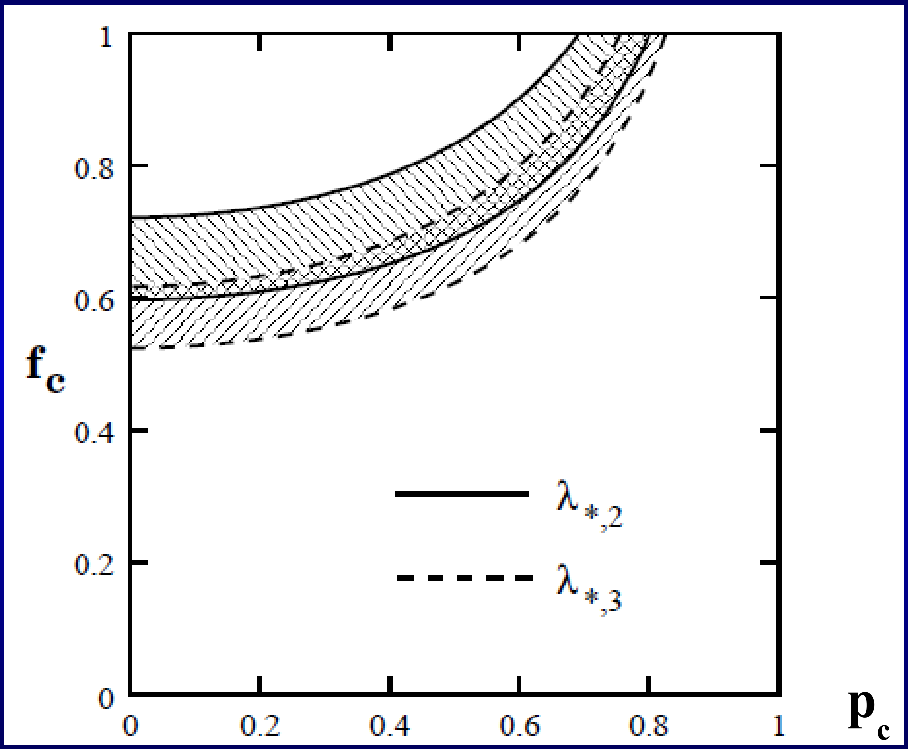
See also

H.C. Eggers, P. Lipa: Int.J.Mod.Phys.E16:3205-3223,2008

Search for partial coherence

Core-halo fraction f_c and partially coherent fraction p_c both

Simultaneous fit to 2nd and 3rd order correlation functions



Hep-ph/0001233:

Analysis of NA44 S+Pb data:

Higher order correlations

Restrict the p_c fraction better.

Dominant halo (f_c < 0.5)

~ full coh. $p_c > 0.8$ excluded

f_c	p_c	$\lambda_{*,2}$	$\lambda_{*,3}$	$\lambda_{*,4}$	$\lambda_{*,5}$
0.60	0.00	0.36	1.51	5.05	17.17
0.70	0.50	0.37	1.45	4.25	11.87
1.00	0.75	0.44	1.63	4.33	10.47

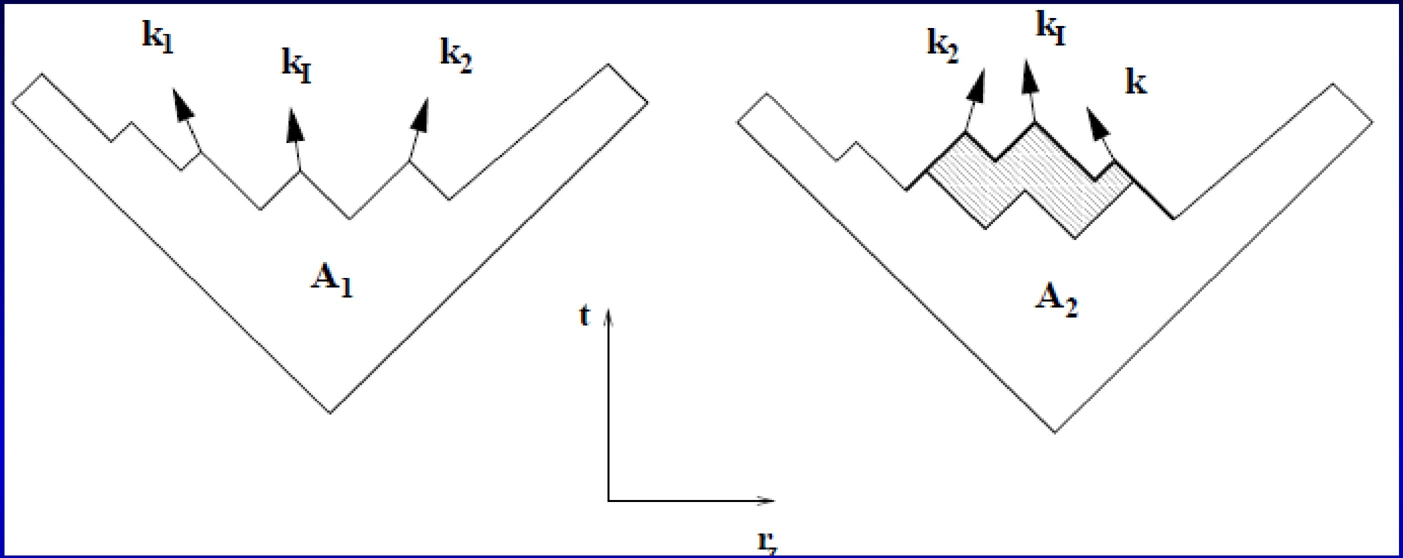
$$\lambda_{*,2} = f_c^2[(1-p_c)^2 + 2p_c(1-p_c)],$$

$$\lambda_{*,3} = 3f_c^2[(1-p_c)^2 + 2p_c(1-p_c)] + 2f_c^3[(1-p_c)^3 + 3p_c(1-p_c)^2],$$

Similar analysis:
L3, NA22, Biyajima...

Andersson-Hoffmann model

Applied in e+e- reactions:

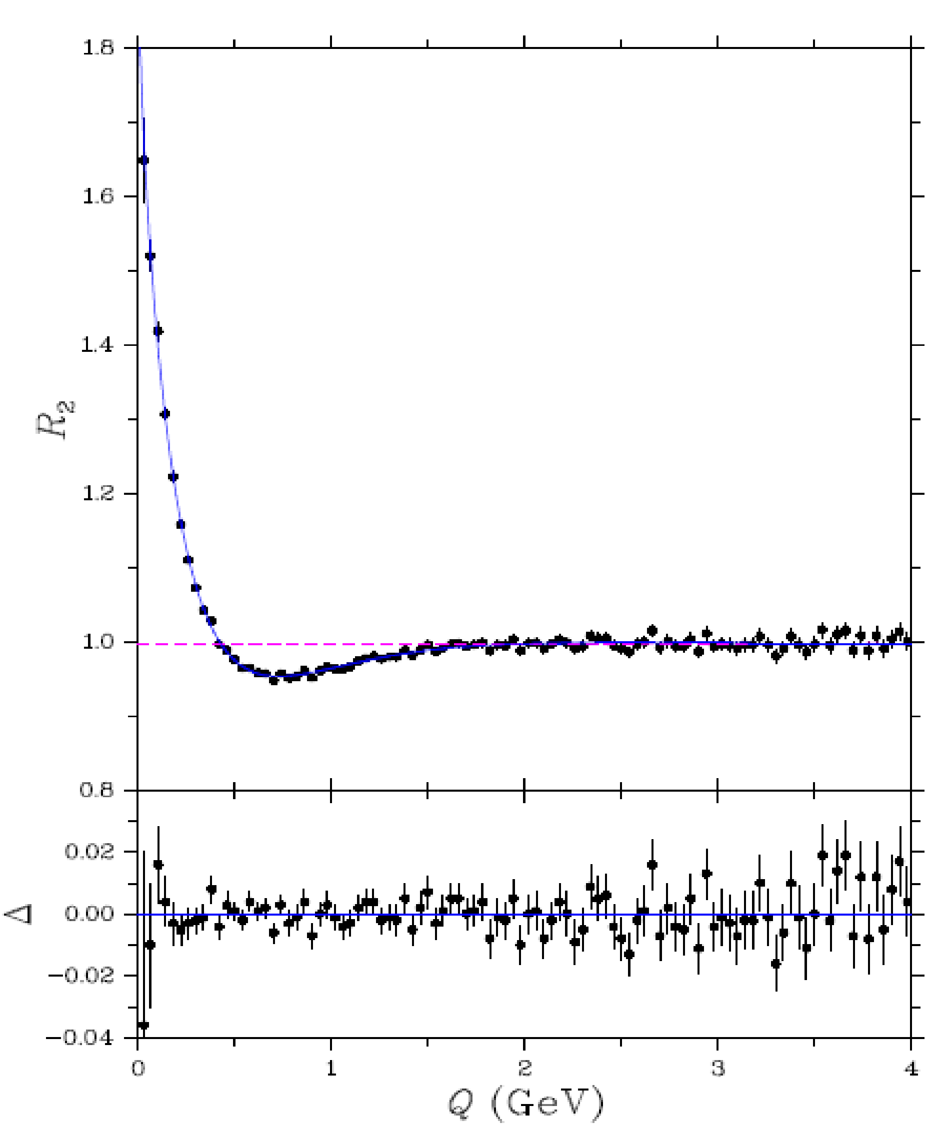


$$M \sim \exp[i(\kappa + ib/2)A_1] + \exp[i(\kappa + ib/2)A_2]\,,$$

$$|\, M \,|^2 \sim [\exp(-bA_1) + \exp(-bA_2)] \cdot \left[1 + \frac{\cos(\kappa \Delta A)}{\cosh(b\Delta A/2)}\right]$$

Suggests: Oscillation (dip), elongation of the source, approx Qinv

Recent L3 results:



Recent L3 result:
dip is significant

Confirms
Earlier TASSO result

Is it only in e^+e^-?

For more details:

See W. Metzger's talk

Several interesting similarities

- **Multiplicity dependence:**
  - **R decreases λ increases with $dn/d\eta$**
- **Transverse mass dependence**
  - **R decreases with increasing m or m_t**
- **Correlations are apparently non-Gaussian**
  - **But in 3 d it is difficult to see the peak**
- **Even for 1 + pos def forms oscillations**
  - **If the source has a binary structure**
- **In e^+e^- collisions, a space-time movie can be**
  - **Recorded $\rightarrow$ not yet possible in h+p, AA**
- **Expanding, non-thermal rings in e^+e^-**
- **Expanding rings of fire seen in h+p reactions**
- **Long, boomerang like shape seen in h+p and e^+e^-**

Metareview

- **B. Lörstad,**
  - **Int.J.Mod.Phys.A4:2861, 1989**
- **W. A. Zajc,**
  - **NATO Adv.Study Inst.Ser.B Phys.303:435-459,1993**
- **M. Lisa, S. Pratt, , R. Soltz, U. A. Wiedemann**
  - **Ann.Rev.Nucl.Part.Sci.55:357-402,2005**
- **T. Cs.**
  - **hep-ph/0001233**
  - **J.Phys.Conf.Ser.50:259, 2006**
- **R. M. Weiner**
  - **Phys.Rept.327:249-346,2000**
- **U. Heinz and U. A. Wiedemann**
  - **Phys.Rept.319:145-230,1999**
- **W. Kittel,**
  - **AIP Conf.Proc.828:519-524,2006 – beyond Gaussian**
- **W. Kittel,**
  - **Hep-ph/9905394 – a critical (p)review**
- **M. G. Bowler: Are the observed Bose-Einstein correlations possible?**
  - **Marburg LESIP IV 1990:2-15 (QCD161:I972:1990)**

WPCF story

  - HBT'96, Trento, Italy (U.Heinz, Cs.T, W.A.Zajc)
  - Warsaw meetings (J. Pluta)
  - Quark Matter 2005 (P. Lévai, Cs.T.)
- WPCF 2005, Kromeriz, Czech Republic
  - M. Sumbera
- WPCF 2006, Sao Paulo, Brazil
  - S. Padula
- WPCF 2007, Livermore, CA, USA
  - R. Soltz
- WPCF 2008, Cracow, Poland
  - A. Bialas
- WPCF 2009, CERN, Switzerland
  - A. Kisiel
- WPCF 2010, Kiev, Ukraine
  - Yu. Sinyukov

Interesting new directions

Azimuthally sensitive HBT (STAR, PHENIX)
Source imaging (PHENIX, STAR)
Multiparticle correlations (STAR, PHENIX)
Non-identical correlations (STAR)
Rapidity dependent HBT (PHOBOS)
Photon HBT (STAR, PHENIX)
Non-Gaussian form (L3, PHENIX, STAR, ALICE, CMS)

- **S. Hegyi, T. Cs., W. A. Zajc, L3, STAR, ...**

Pion lasers

- **S. Pratt, Q.H. Zhang, T. Cs, J. Zimányi, Yu. Sinyukov...**

Mass-modification, squeezing

- **M. Asakawa, T. Cs., M. Gyulassy, Y. Hama, S. Padula, ...**

Search for axial $U_A(1)$ symmetry restoration, $\lambda(p_t)$

- **S. Vance, T. Cs., D. Kharzeev, R. Vértesi, J. Sziklai**