

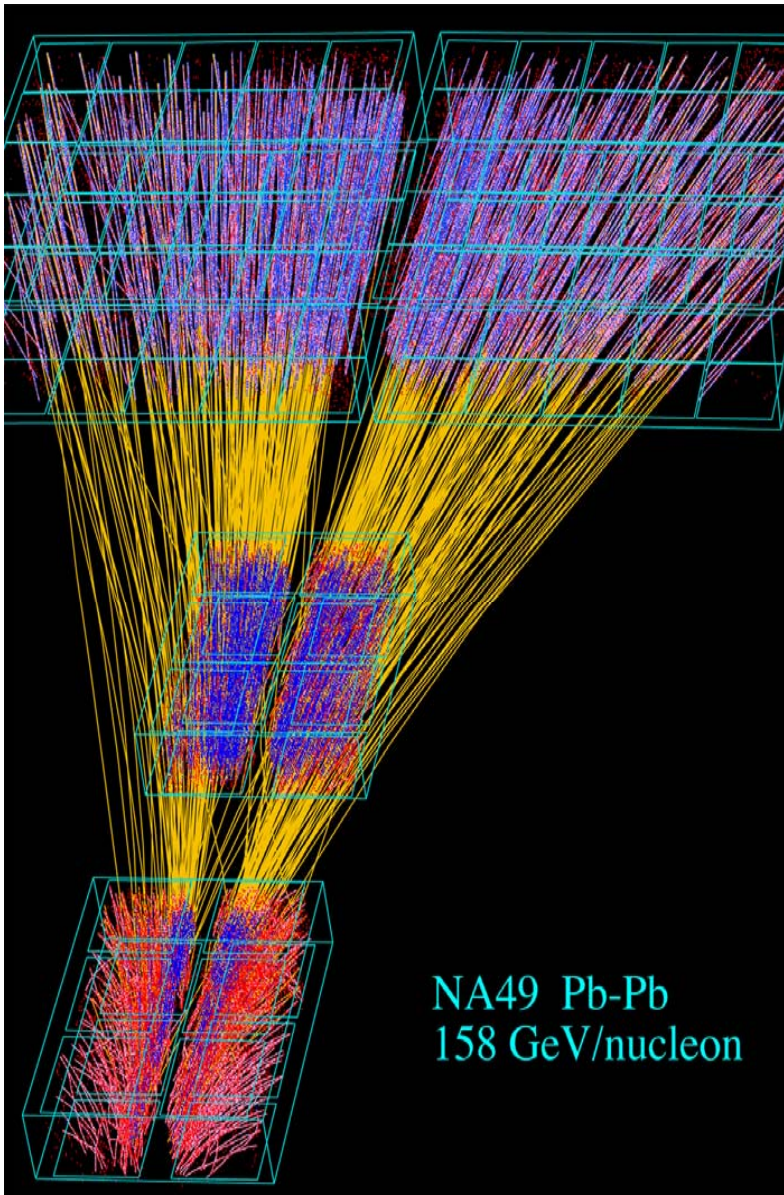
# Fluctuations and Correlations in Statistical Models

Mark I. Gorenstein

Bogolyubov Institute for Theoretical Physics  
Kiev, Ukraine

1. Statistical Ensembles
2. Particle Number Fluctuations
3. New Concept of Statistical Ensembles
4. MCE/sVF:
  - a) KNO-scaling;
  - b) power-law in  $p_T$





$$N = 10^2 \div 10^4$$

$$P(N), \quad \langle N^k \rangle = \sum_N N^k P(N)$$

$$\begin{aligned} \text{Var}(N) &= \langle N^2 \rangle - \langle N \rangle^2 \\ &= \langle (N - \langle N \rangle)^2 \rangle = \langle (\Delta N)^2 \rangle \end{aligned}$$

$$\omega = \frac{\text{Var}(N)}{\langle N \rangle}$$

Scaled Variances are not equal to each other in different SE

# GCE and CE

$$Z_{gce} = \sum_{N_+, N_- = 0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} = \exp(2z)$$

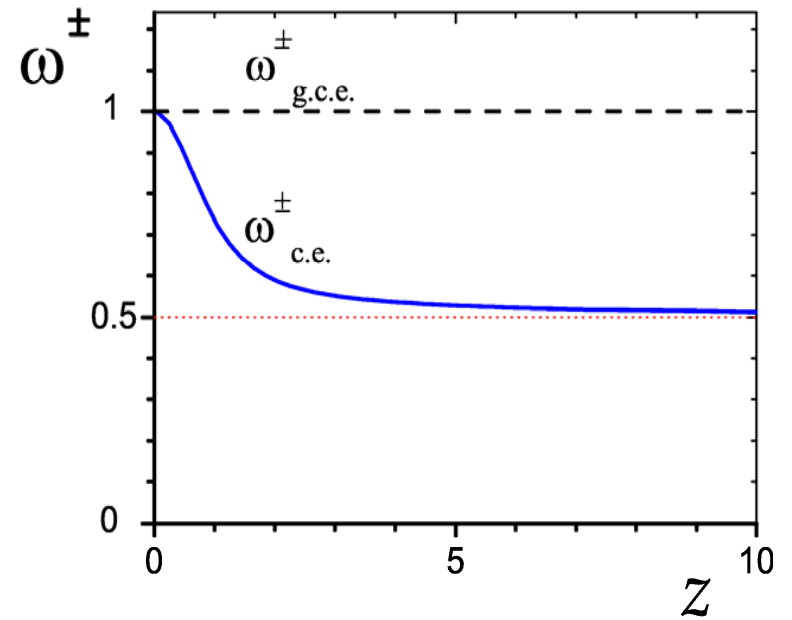
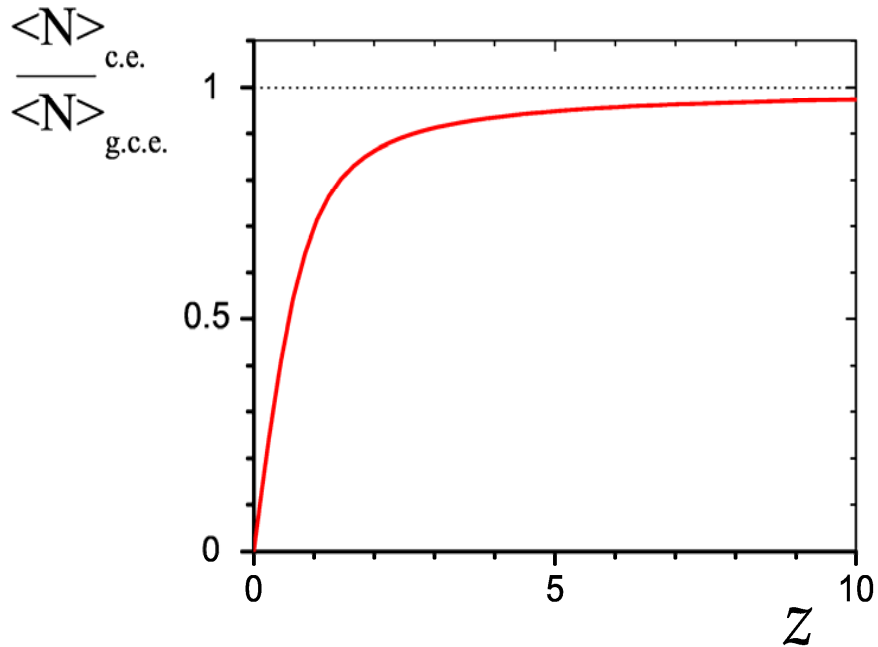
$$z = \frac{V}{2\pi^2} T m^2 K_2(m/T)$$

$$Z_{ce} = \sum_{N_+, N_- = 0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} \delta(N_+ - N_-) = I_0(2z)$$

$$\omega^- = \frac{\langle N_-^2 \rangle - \langle N_- \rangle^2}{\langle N_- \rangle}, \quad \langle N_- \rangle_{gce} = z, \quad \omega_{gce}^- = 1$$

$$\langle N_- \rangle_{ce} = z \frac{I_1(2z)}{I_0(2z)}, \quad \omega_{ce}^- = 1 - z \left[ \frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)} \right]$$

# GCE and CE



**Begun, Gazdzicki, M.I.G., Zozulya  
Phys. Rev. C (2004)**

# Statistical Ensembles $E, V, Q$

$$E \longleftrightarrow T$$

$$E, V, Q \quad \text{MCE}$$

$$V \longleftrightarrow p$$

$$T, V, Q \quad \text{CE}$$

$$Q \longleftrightarrow \mu_Q$$

$$T, V, \mu_Q \quad \text{GCE}$$

$$2^3 = 8$$

$$E, V, \mu_Q \quad \text{MGCE}$$

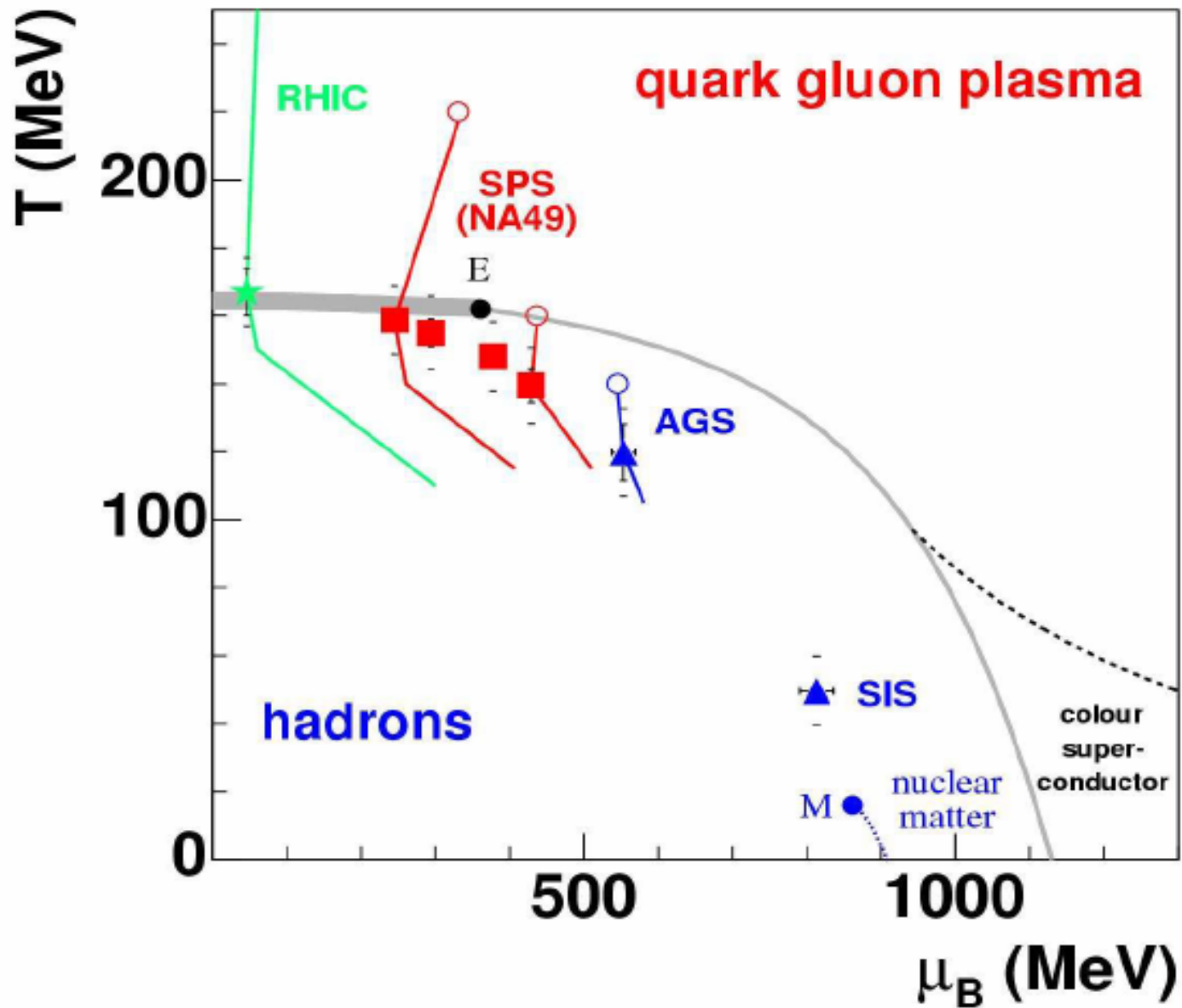
$$E, p, Q \quad T, p, Q$$

$$T, p, \mu_Q \quad E, p, \mu_Q$$

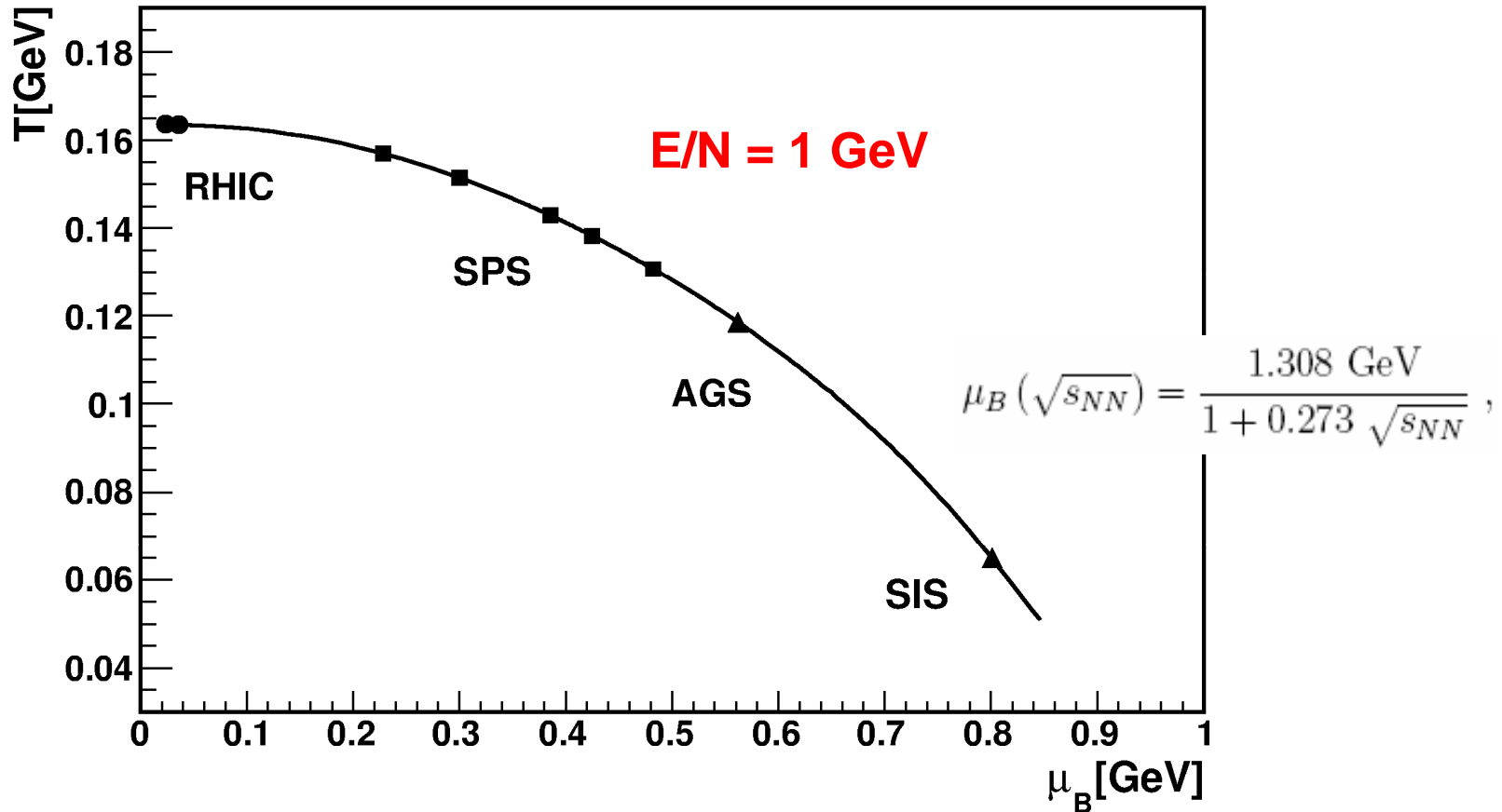
Gorenstein,  
J. Phys. G  
(2008)

Pressure

Ensembles



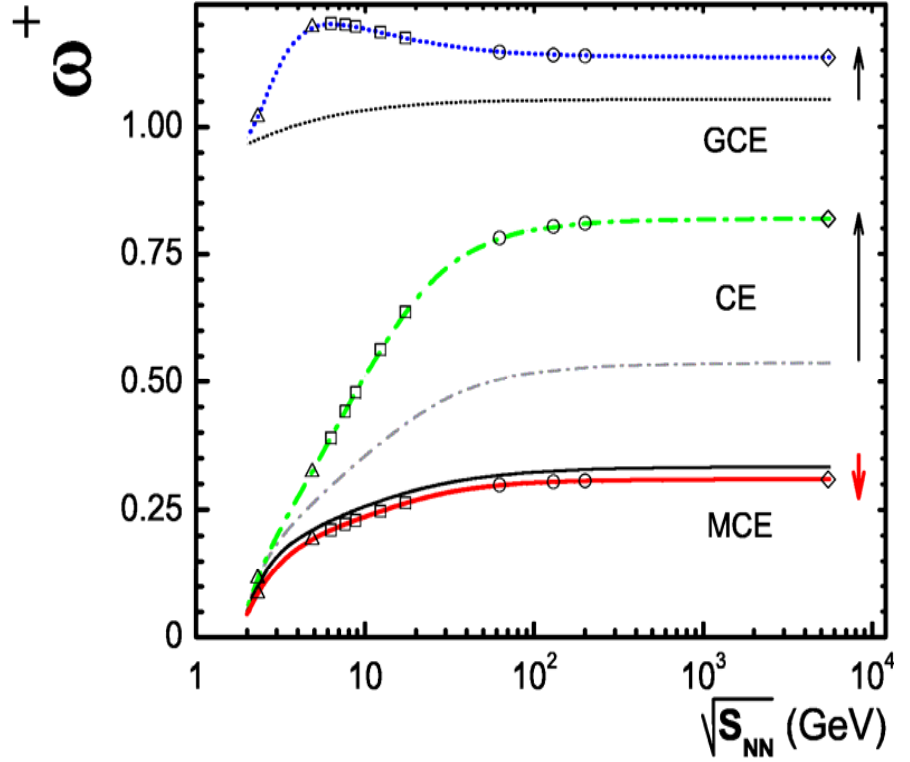
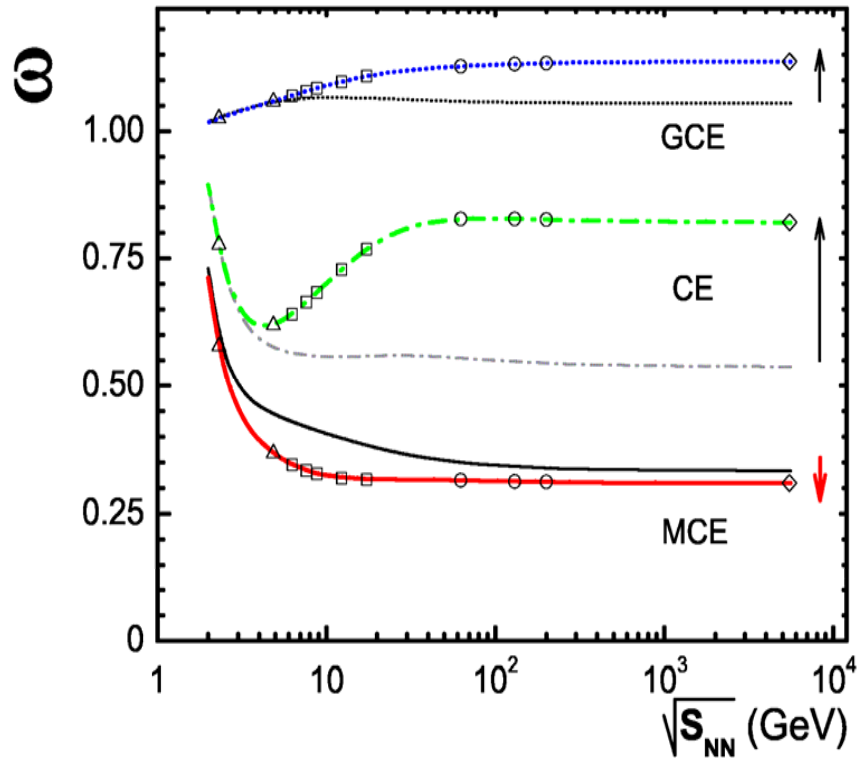
# Line of the chemical freeze-out



Cleymans and Redlich, Phys. Rev. Lett. (1998)



# The prediction of hadron gas model

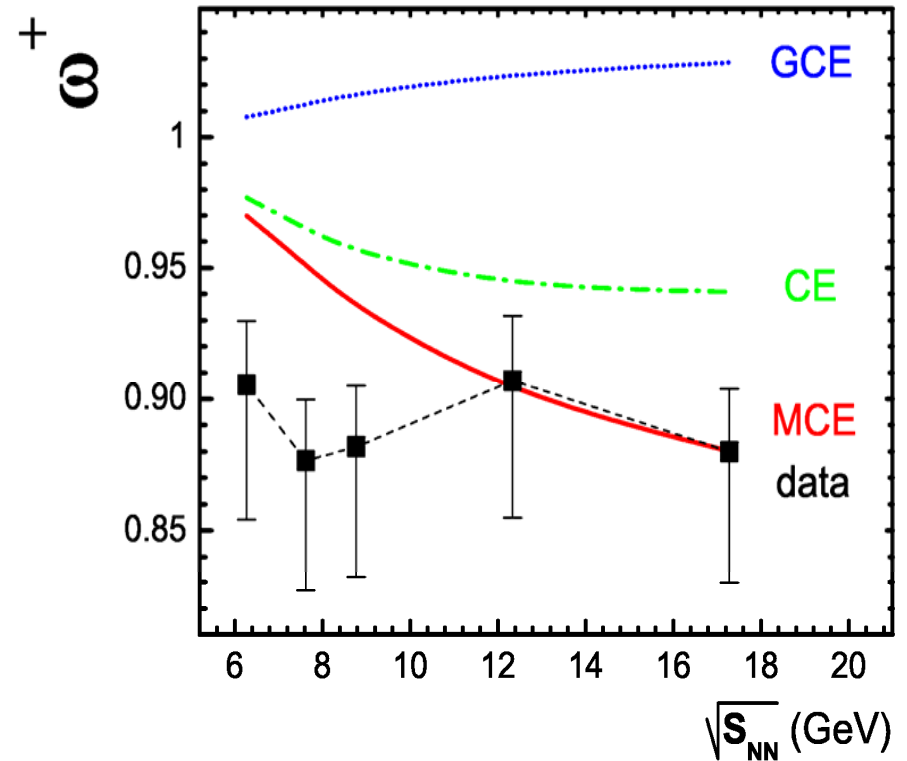
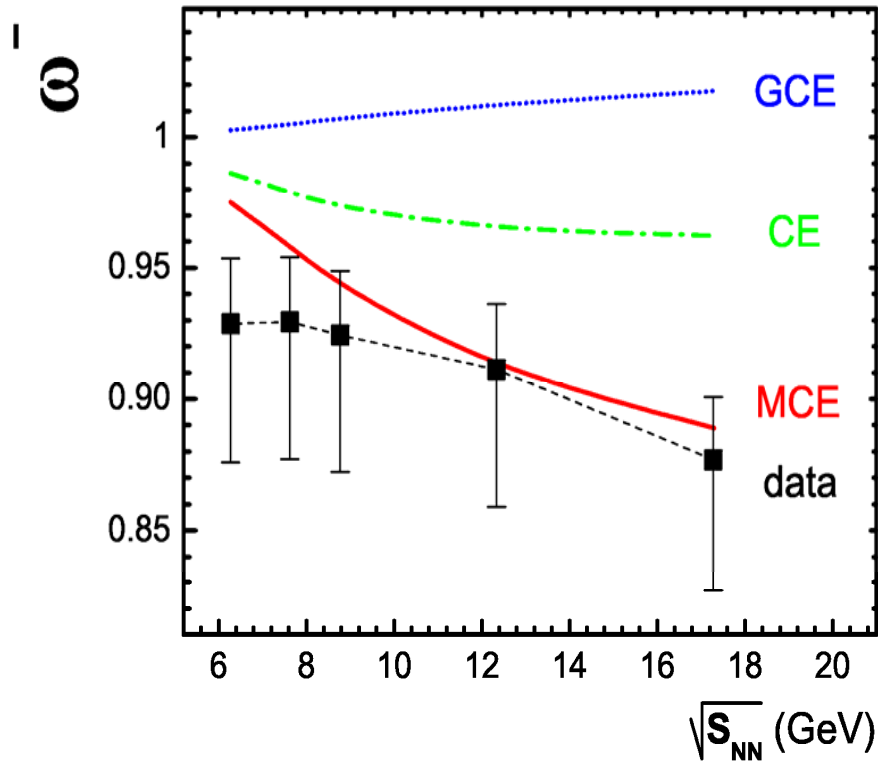


$$\omega_{4\pi}^{\pm} \equiv \frac{\langle (\Delta N_{\pm})^2 \rangle}{\langle N_{\pm} \rangle}$$

$$\omega_{acc}^{\pm} = 1 - q + q \omega_{4\pi}^{\pm}$$

**Begun, Gazdzicki, M.I.G. Hauer, Konchakovski, Lungwitz,  
Phys. Rev. C (2007)**

# Comparison with the NA49 data



$$q = 0.038, 0.063, 0.085, 0.131, 0.163$$

Begun, Gazdzicki, M.I.G. Hauer, Konchakovski, Lungwitz,  
Phys. Rev. C (2007)

$$\vec{A} = (E, V, Q_1, \dots, Q_k)$$

## Alpha-Enesmbles

$$P_\alpha(X) = \int d\vec{A} P_\alpha(\vec{A}) P_{mce}(X; \vec{A})$$

M.I.G. , Hauer, Phys. Rev. C (2008)

# Problems of the statistical approach

## I. “Non-Statistical” Multiplicity Distributions in

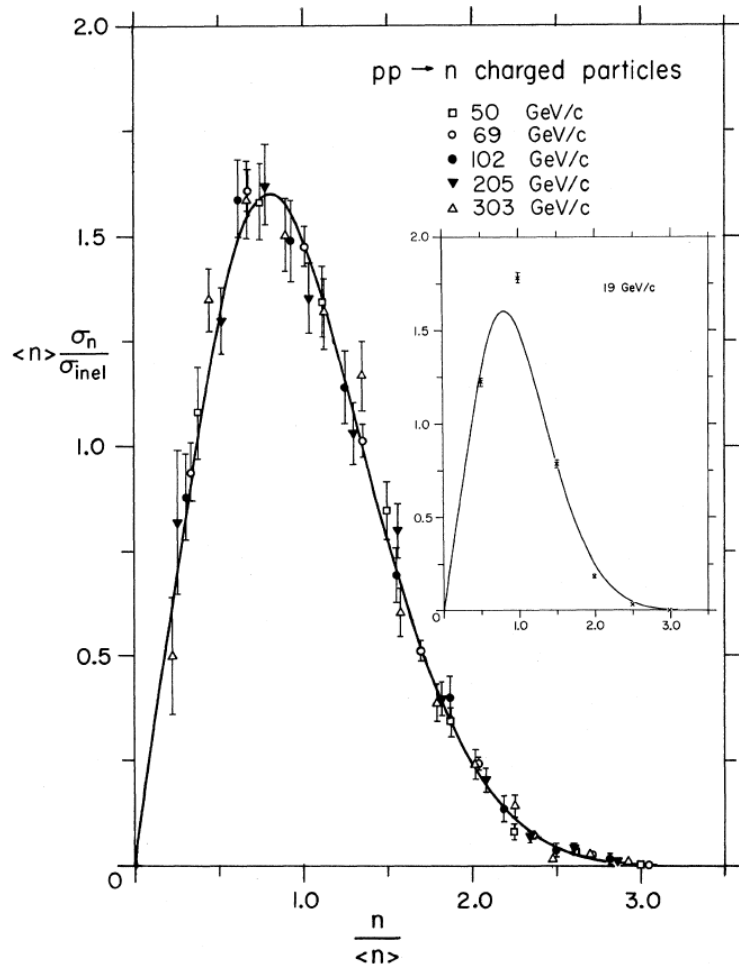
$$e^+e^-, pp, p\bar{p}$$

## II. Power law at high $p_T$ and high $m$

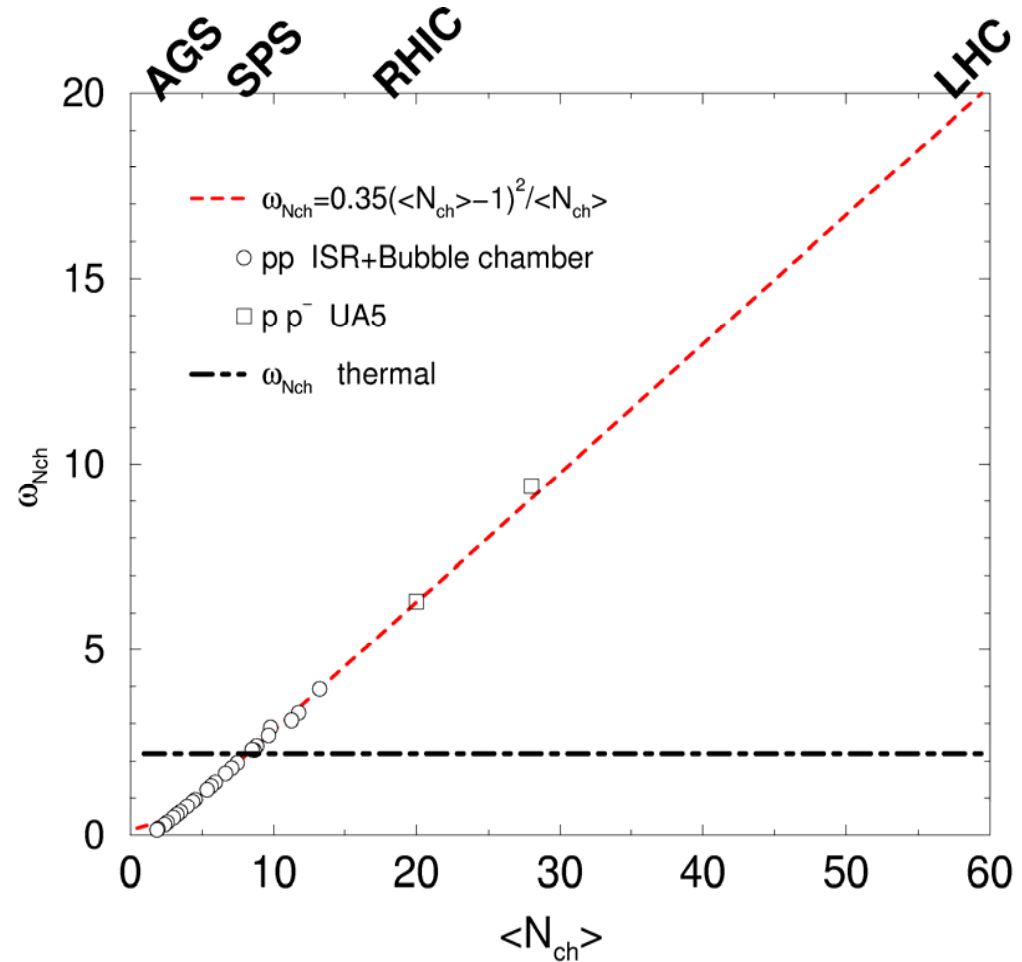
$$\frac{d^3N_i}{dp^3} \sim C p_T^{-K_p} \quad p_T \gg m_i$$

$$\langle N_i \rangle \sim C m_i^{-K_m} \quad \begin{array}{l} K_p \approx 8 \\ K_m \approx K_p - 3 \end{array}$$

# KNO scaling & Large fluctuations



Slattery, Phys. Rev. Lett. (1972);



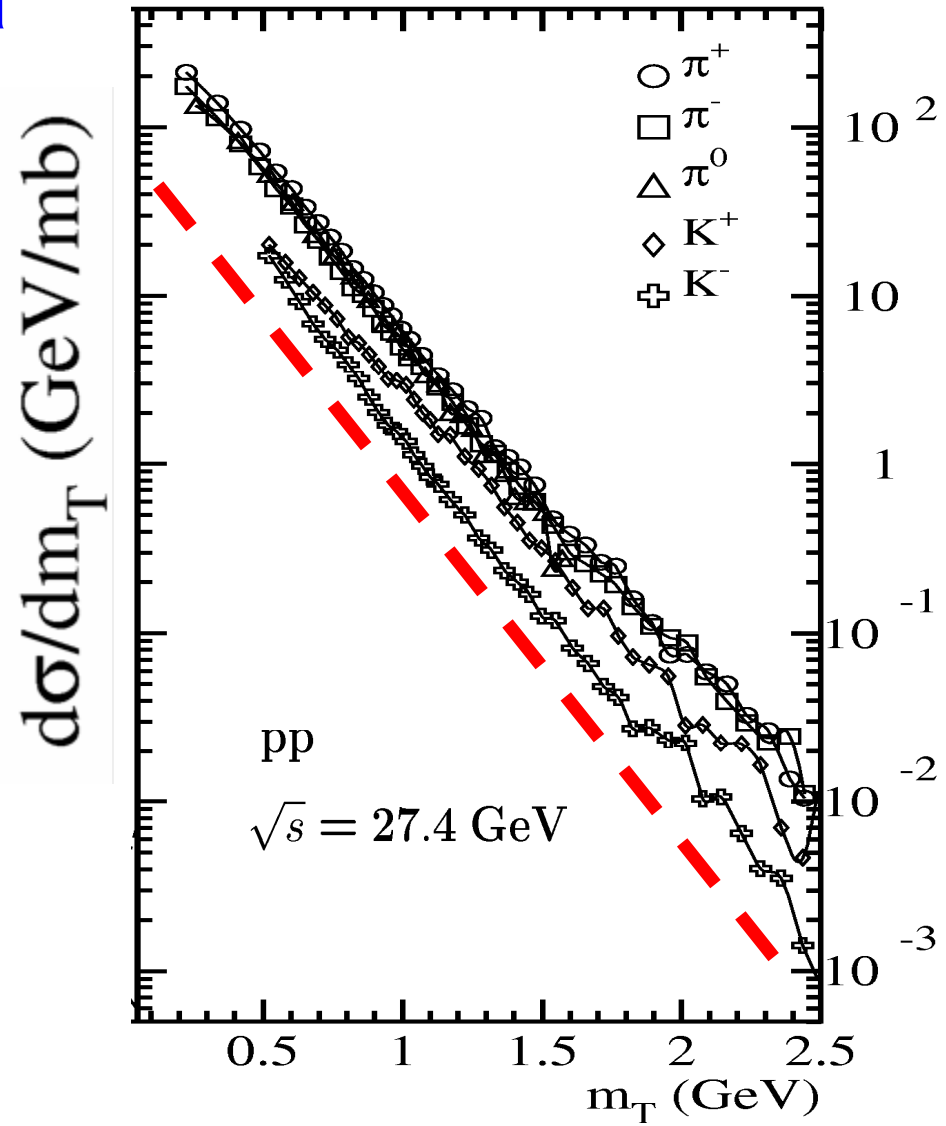
Heiselberg, Phys. Rept. (2001)

# Momentum Spectra

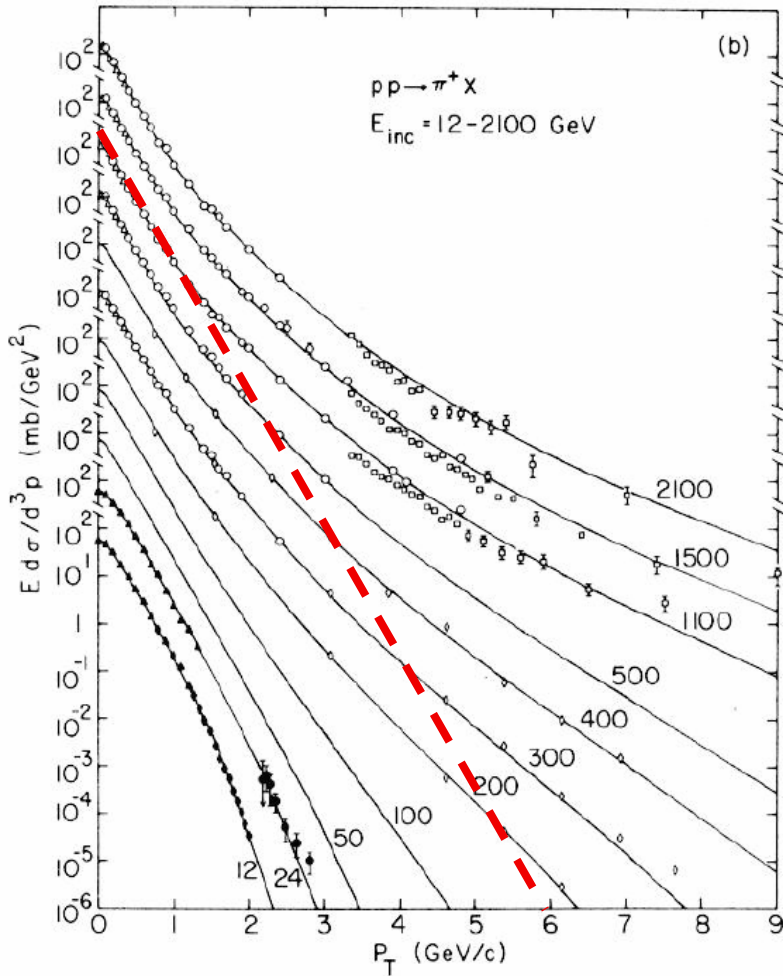
Becattini, Passaleva,  
Eur. Phys. J. (2002),  
data from  
Aguilar-Benitez et al.,  
Z. Phys. C (1990)

$$\exp\left(-\frac{m_T}{T}\right)$$

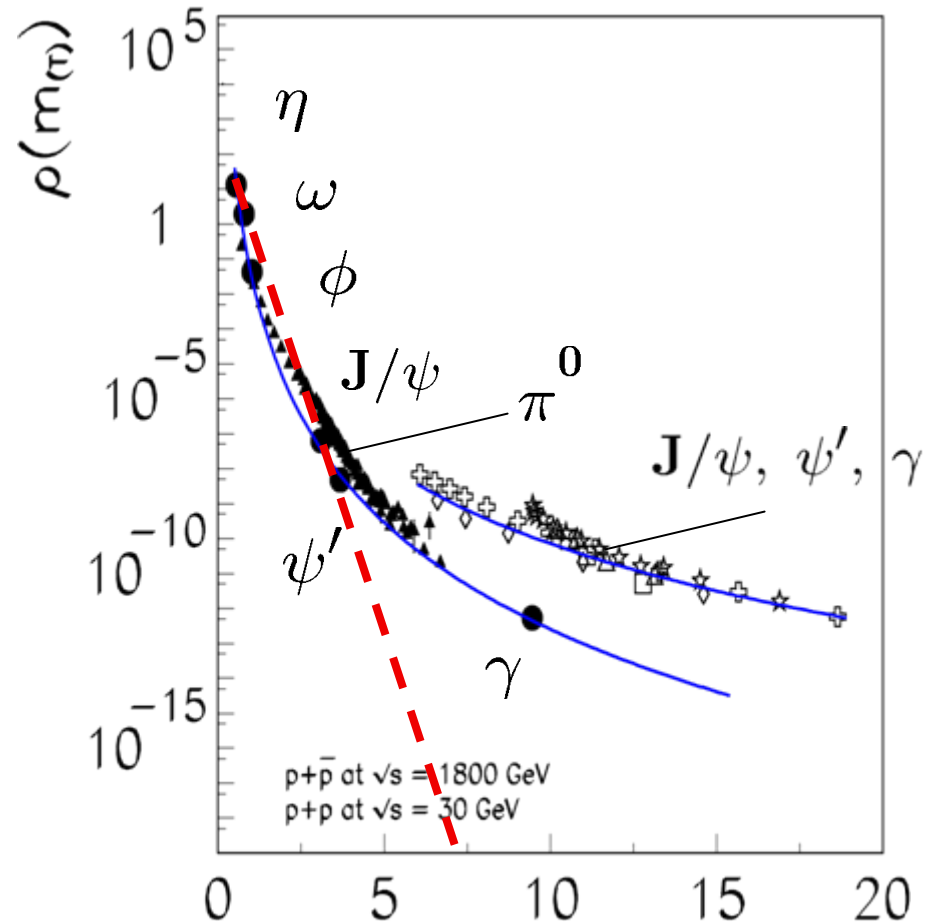
$$m_T = \sqrt{p_T^2 + m^2}$$



# Power law at high $p_T$ and high $m$



Beier *et al.*, Phys. Rev. D (1978)



Gazdzicki, Gorenstein,  
 Phys. Lett. B (2001)

$m(m) [\text{GeV}/c^2]$

# I. Multiplicity distribution

## KNO scaling & Large fluctuations

**Data:** 
$$P(N) = \frac{1}{\langle N \rangle} \Psi_\alpha \left( \frac{N}{\langle N \rangle} \right),$$
 **Koba, Nielsen, Olesen, Nucl. Phys. B (1972)**

$$\omega \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} \propto \langle N \rangle$$

**Statistical Models:**

$$P(N) \cong \frac{1}{\sqrt{2\pi\omega\langle N \rangle}} \exp \left[ -\frac{(N - \langle N \rangle)^2}{2\omega\langle N \rangle} \right],$$

$$\omega \approx \text{const} \approx 1$$



# Micro Canonical Ensemble with scaling Volume Fluctuations (MCE/sVF)

$$P_{\alpha}(\mathbf{X}; \mathbf{E}) = \int_0^{\infty} dV P_{\alpha}(V) P_{\text{mce}}(\mathbf{X}; \mathbf{E}, V)$$

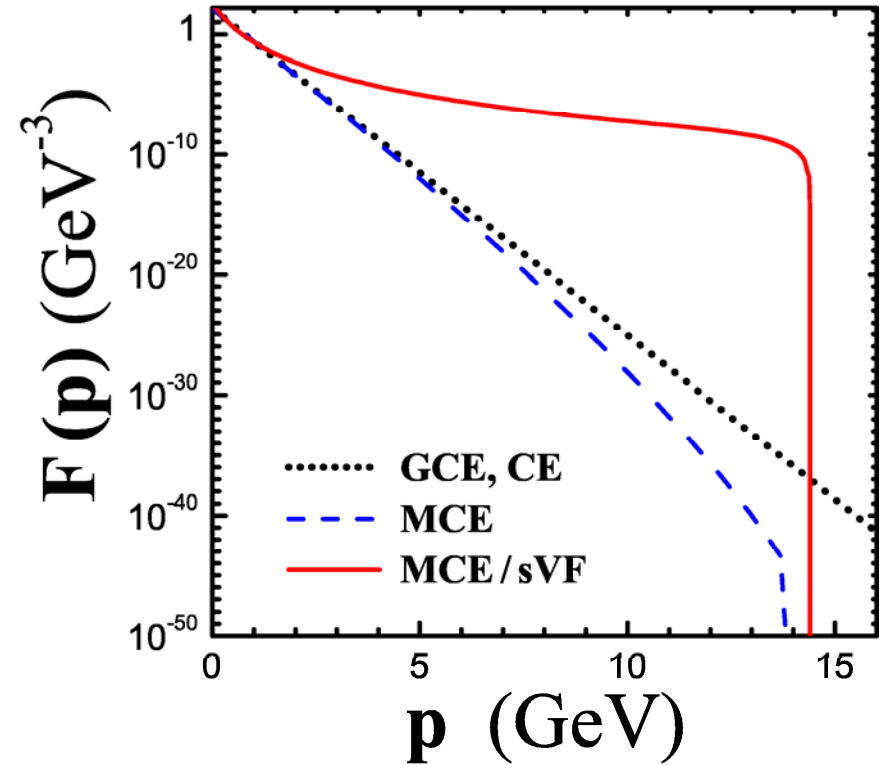
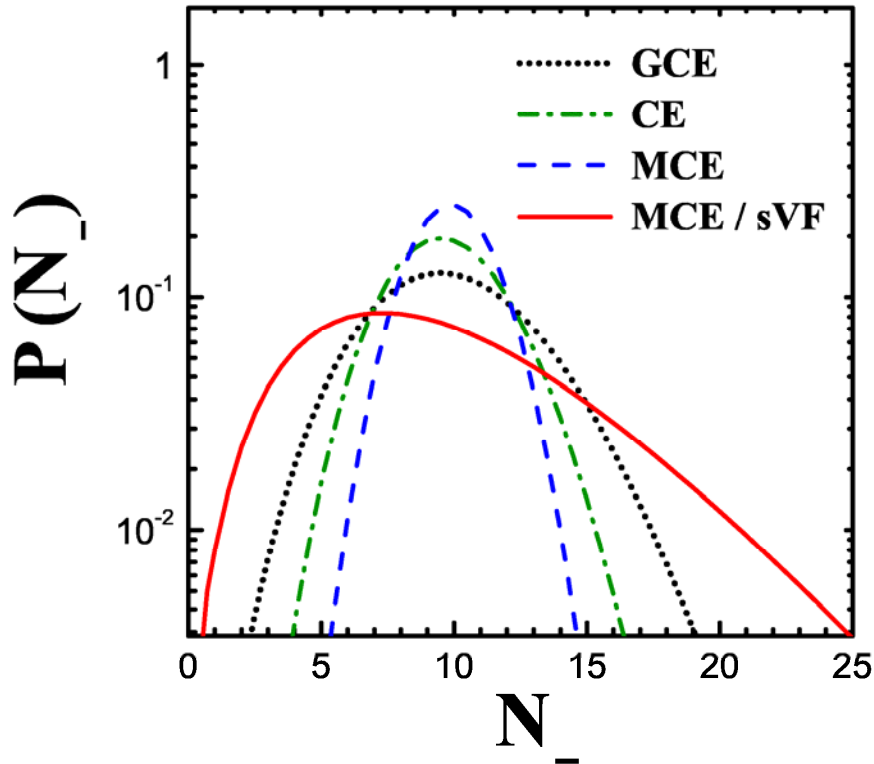
$$\mathbf{X} = \mathbf{N}, p$$

Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008)

$$P_{\alpha}(V) = \frac{1}{\bar{V}} \Phi_{\alpha}(V/\bar{V})$$

Scaling volume fluctuations selected  
to fit **experimental** multiplicity **distribution**

# Particle Number Distributions and Spectra

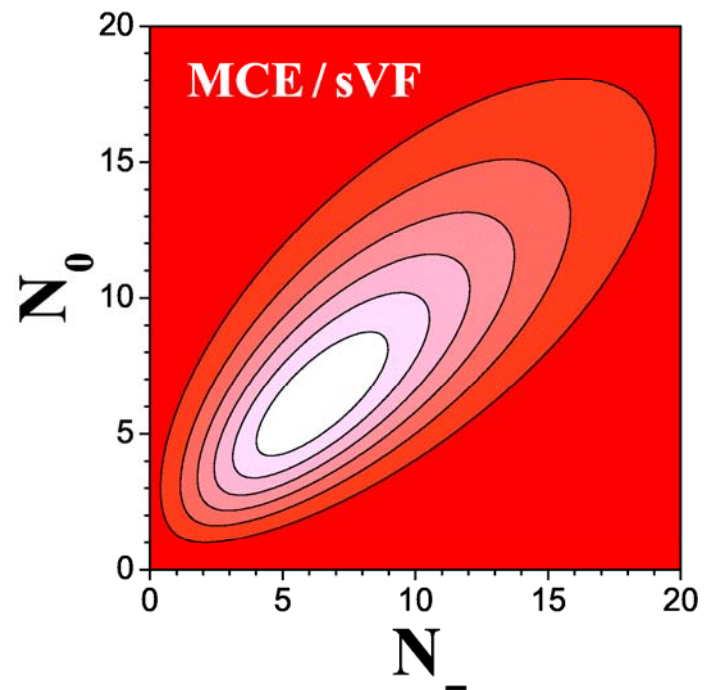
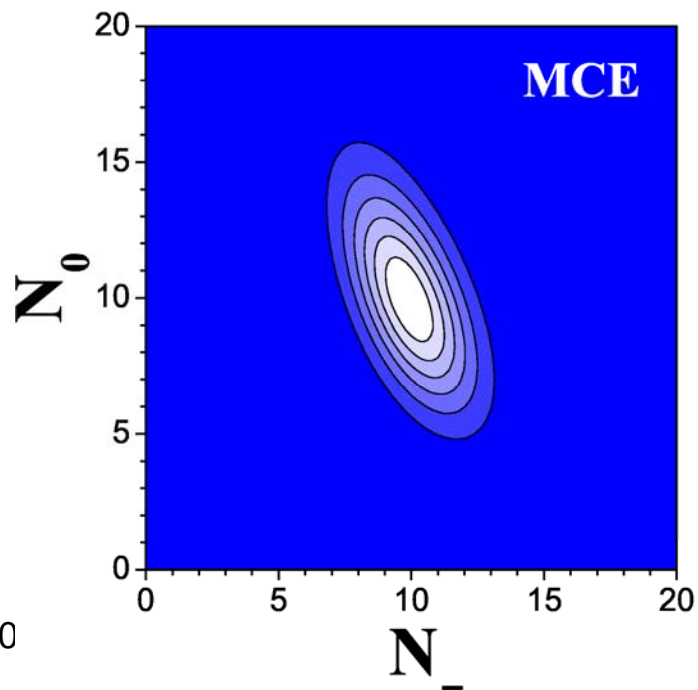
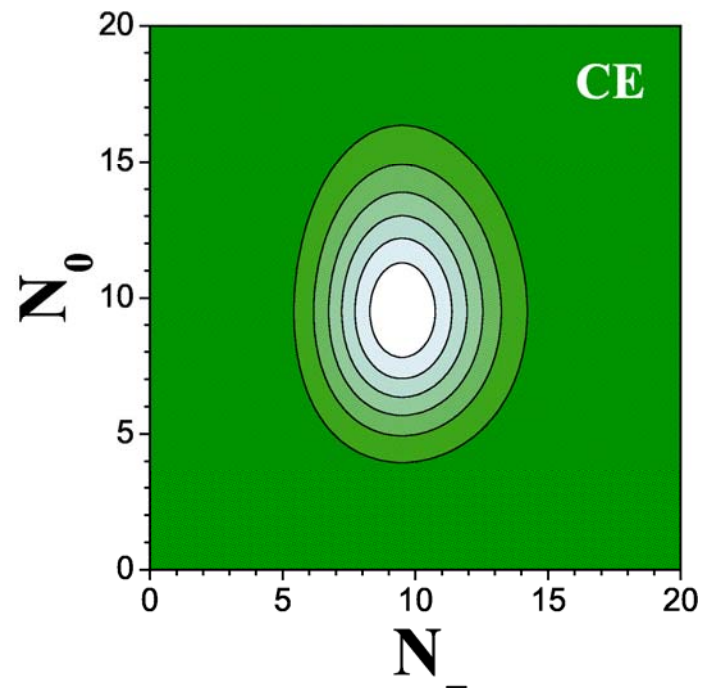
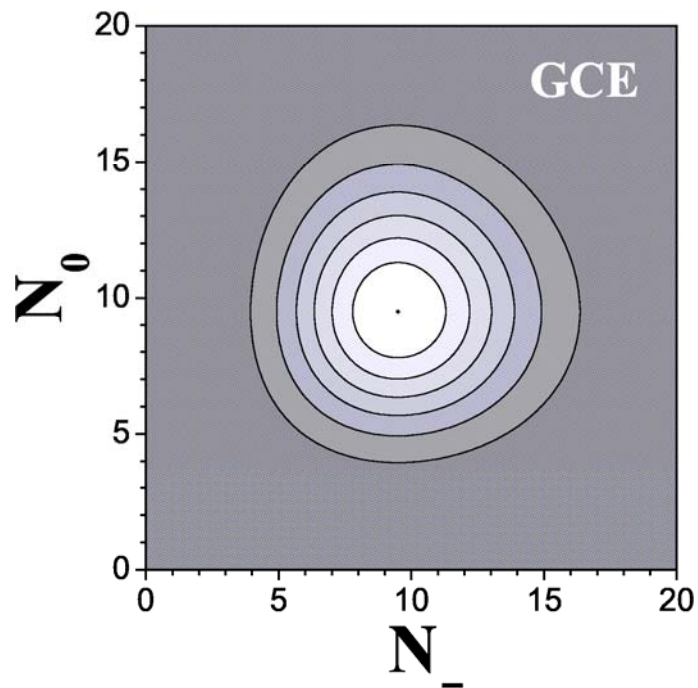


$$P_\alpha(N) = \frac{1}{\bar{N}} \Psi_\alpha(N/\bar{N})$$

$$\Psi_\alpha(y) = \frac{k^k}{3!} y^{k-1} \exp(-ky)$$

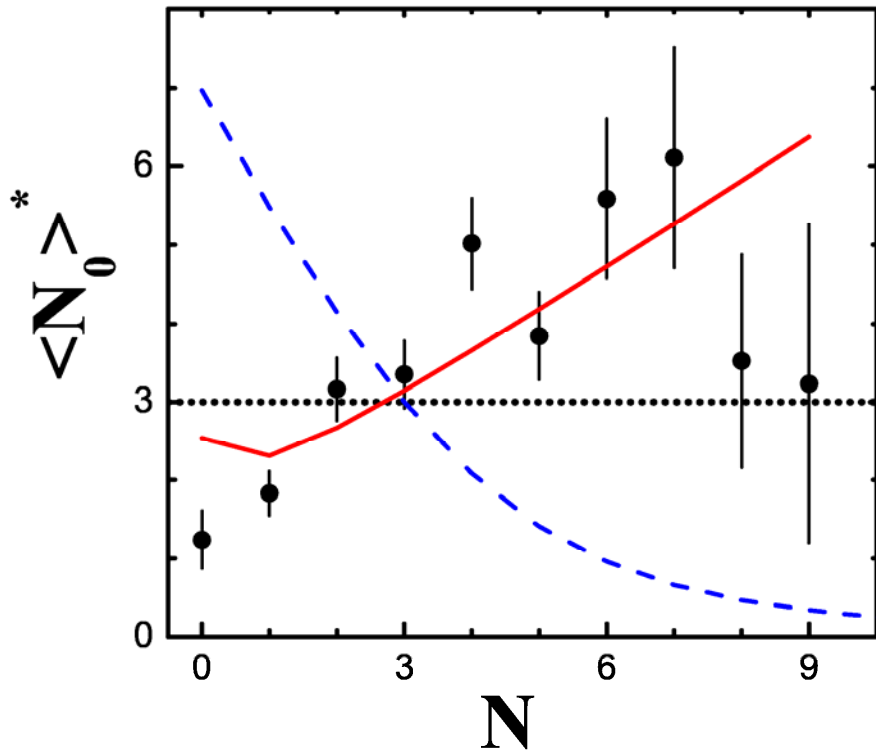
$$F_\alpha(p) \approx \frac{k^k \Gamma(k+4)}{2\Gamma(k)} T^{k+1} (p+kT)^{-k-4}$$

$$\approx 11.27 \text{ GeV}^5 (p+4T)^{-8}$$

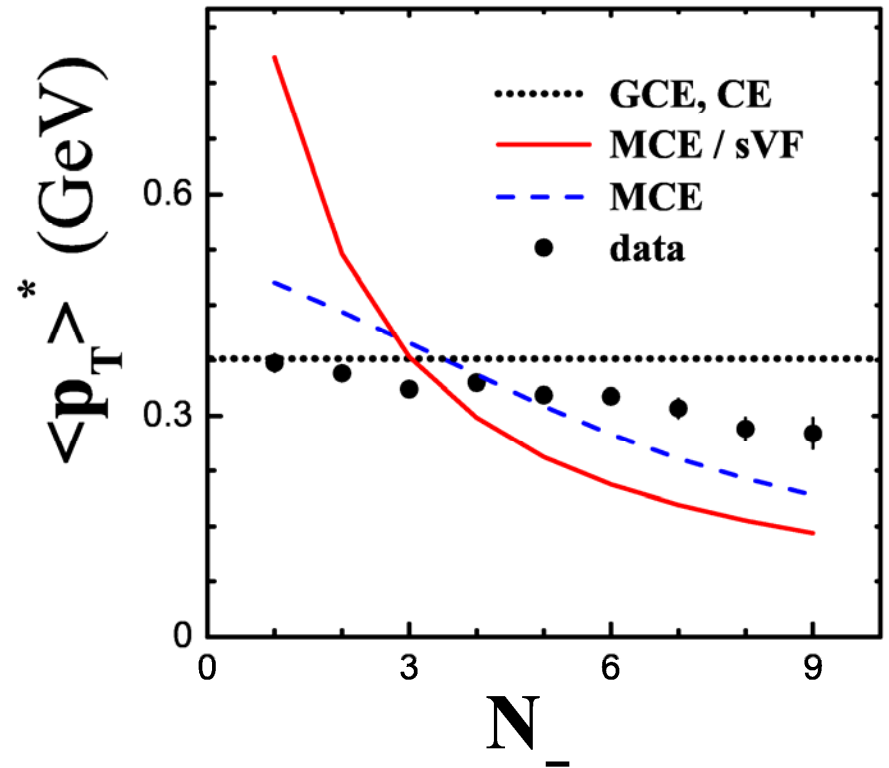


# Semi-Inclusive Observables

Begun, Gazdzicki, M.I.G, Phys. Rev. (2009)



Data on  $p+p$  at 205 GeV/c  
Phys. Rev. D (1975) and (1977)



# Summary

1. **Statistical Ensembles with Fluctuating Extensive Quantities**
2. **MCE/sVF**
  - a) **Large Particle Number Fluctuations**
  - b) **Power Law at Large Transverse Momenta**
3. **Semi-Inclusive Observables in Statistical Mechanics**