

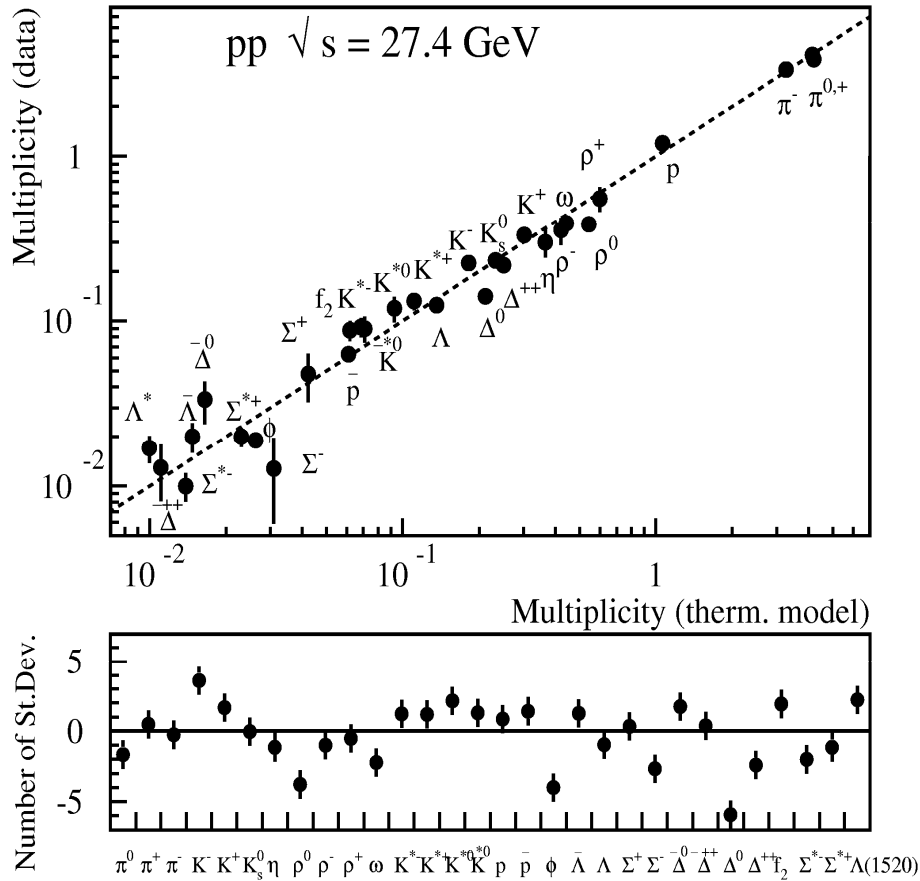
Fluctuations and Correlations in Statistical Models

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1. Statistical Ensembles
2. Particle Number Fluctuations
3. New Concept of Statistical Ensembles
4. MCE/sVF:
 - a) KNO-scaling;
 - b) power-law in p_T

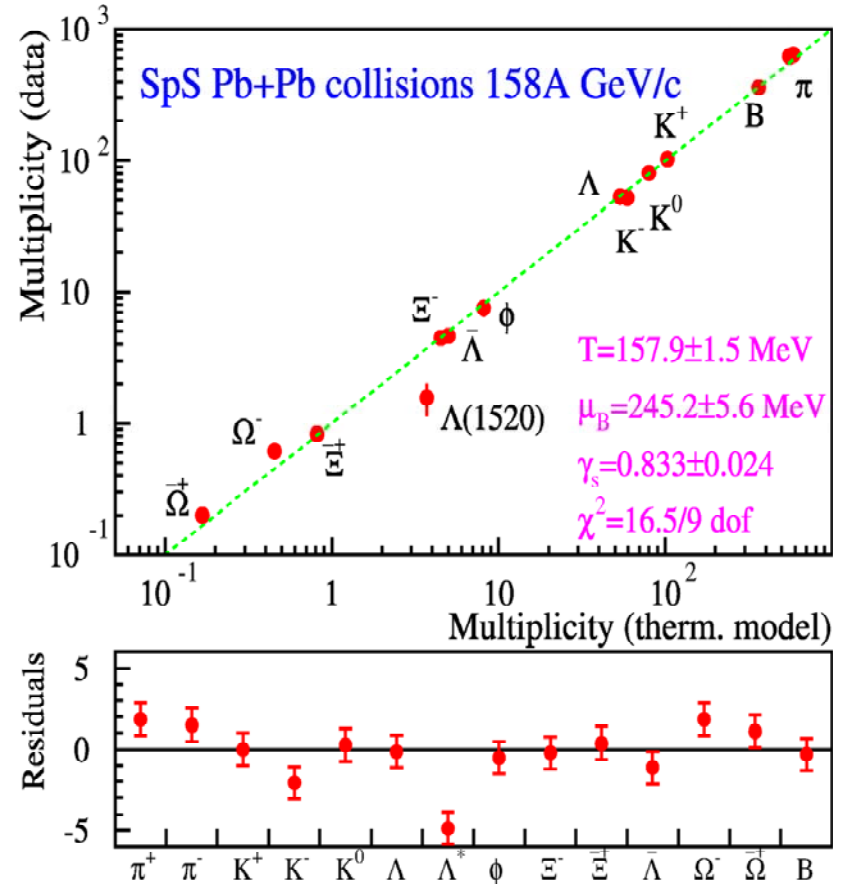
CE Mean Multiplicities GCE



Becattini, Heinz, Z. Phys. (1997) **CE** in p+p

M.I.G., Gazdzicki, Greiner,
Phys. Lett. B (2000)

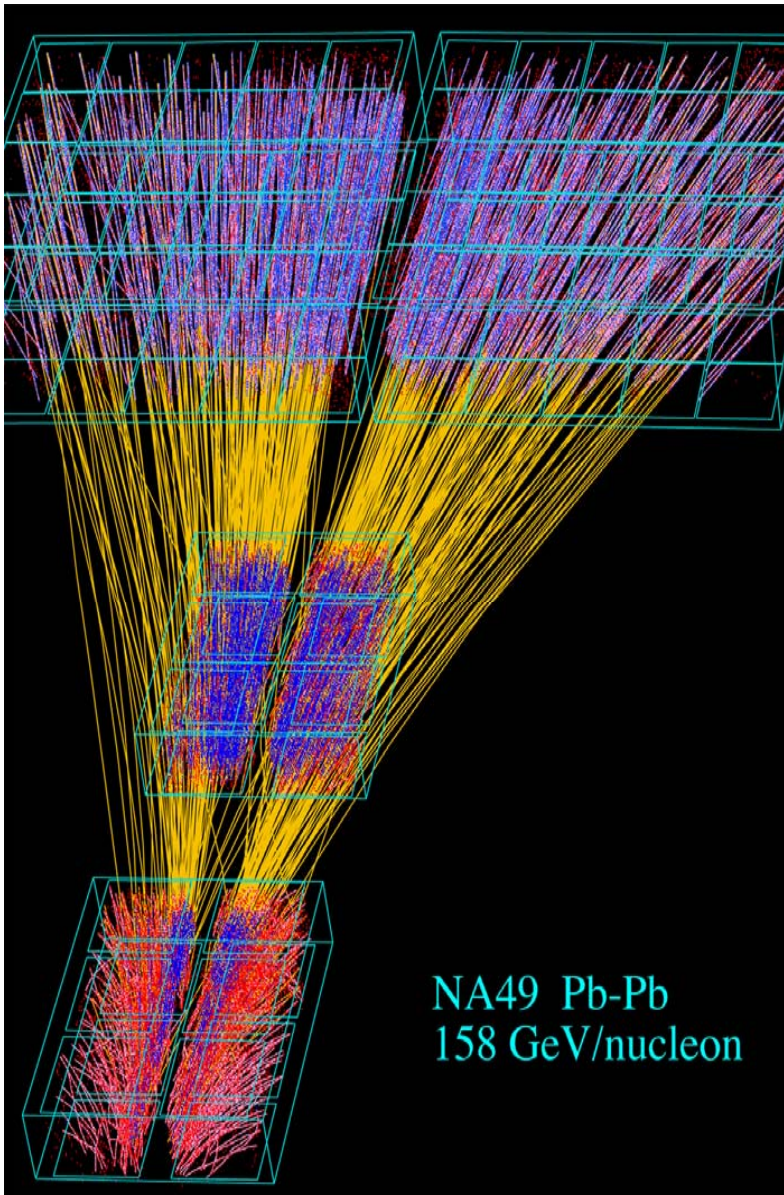
CE for antibaryons in p+A



Becattini, Manninen, Gazdzicki,
Phys. Rev. C (2006) **GCE** in A+A

M.I.G., Kostyuk, Stoecker, Greiner,
Phys. Lett. B (2001)

CE for charmed hadrons



$$N = 10^2 \div 10^4$$

$$P(N), \quad \langle N^k \rangle = \sum_N N^k P(N)$$

$$\begin{aligned} \text{Var}(N) &= \langle N^2 \rangle - \langle N \rangle^2 \\ &= \langle (N - \langle N \rangle)^2 \rangle = \langle (\Delta N)^2 \rangle \end{aligned}$$

$$\omega = \frac{\text{Var}(N)}{\langle N \rangle}$$

Scaled Variances are not equal to each other in different SE

GCE and CE

$$Z_{gce} = \sum_{N_+, N_- = 0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} = \exp(2z)$$

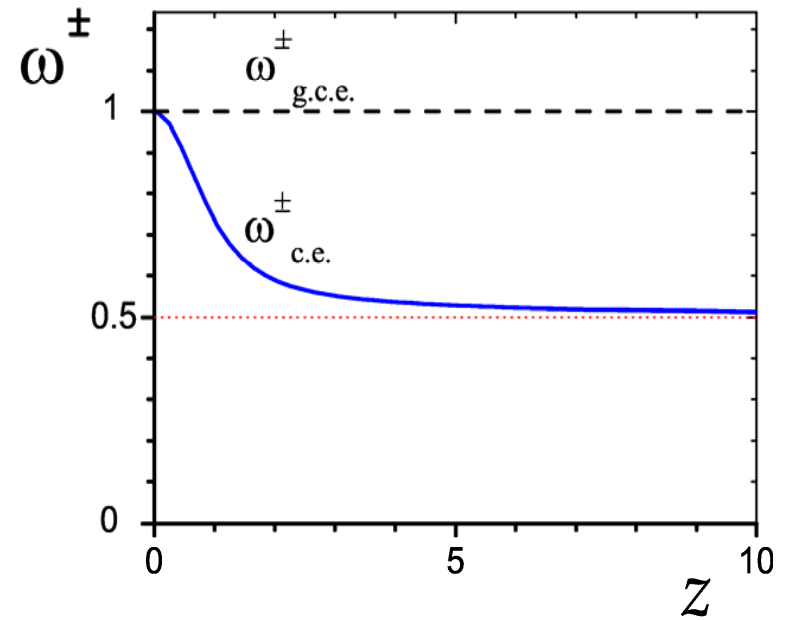
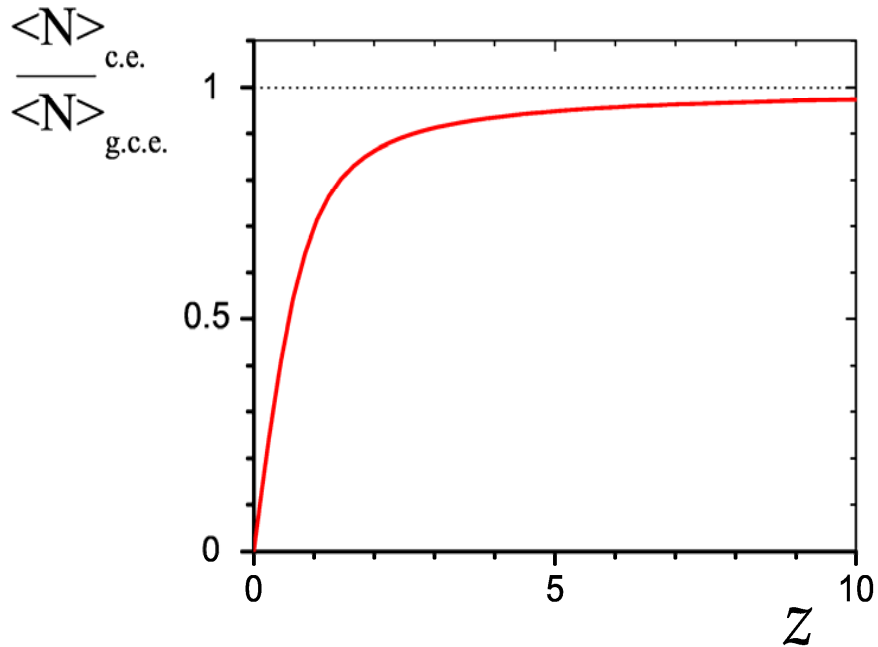
$$z = \frac{V}{2\pi^2} T m^2 K_2(m/T)$$

$$Z_{ce} = \sum_{N_+, N_- = 0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} \delta(N_+ - N_-) = I_0(2z)$$

$$\omega^- = \frac{\langle N_-^2 \rangle - \langle N_- \rangle^2}{\langle N_- \rangle}, \quad \langle N_- \rangle_{gce} = z, \quad \omega_{gce}^- = 1$$

$$\langle N_- \rangle_{ce} = z \frac{I_1(2z)}{I_0(2z)}, \quad \omega_{ce}^- = 1 - z \left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)} \right]$$

GCE and CE



**Begun, Gazdzicki, M.I.G., Zozulya
Phys. Rev. C (2004)**

Statistical Ensembles E, V, Q

$$E \longleftrightarrow T$$

$$E, V, Q \quad \text{MCE}$$

$$V \longleftrightarrow p$$

$$T, V, Q \quad \text{CE}$$

$$Q \longleftrightarrow \mu_Q$$

$$T, V, \mu_Q \quad \text{GCE}$$

$$2^3 = 8$$

$$E, V, \mu_Q \quad \text{MGCE}$$

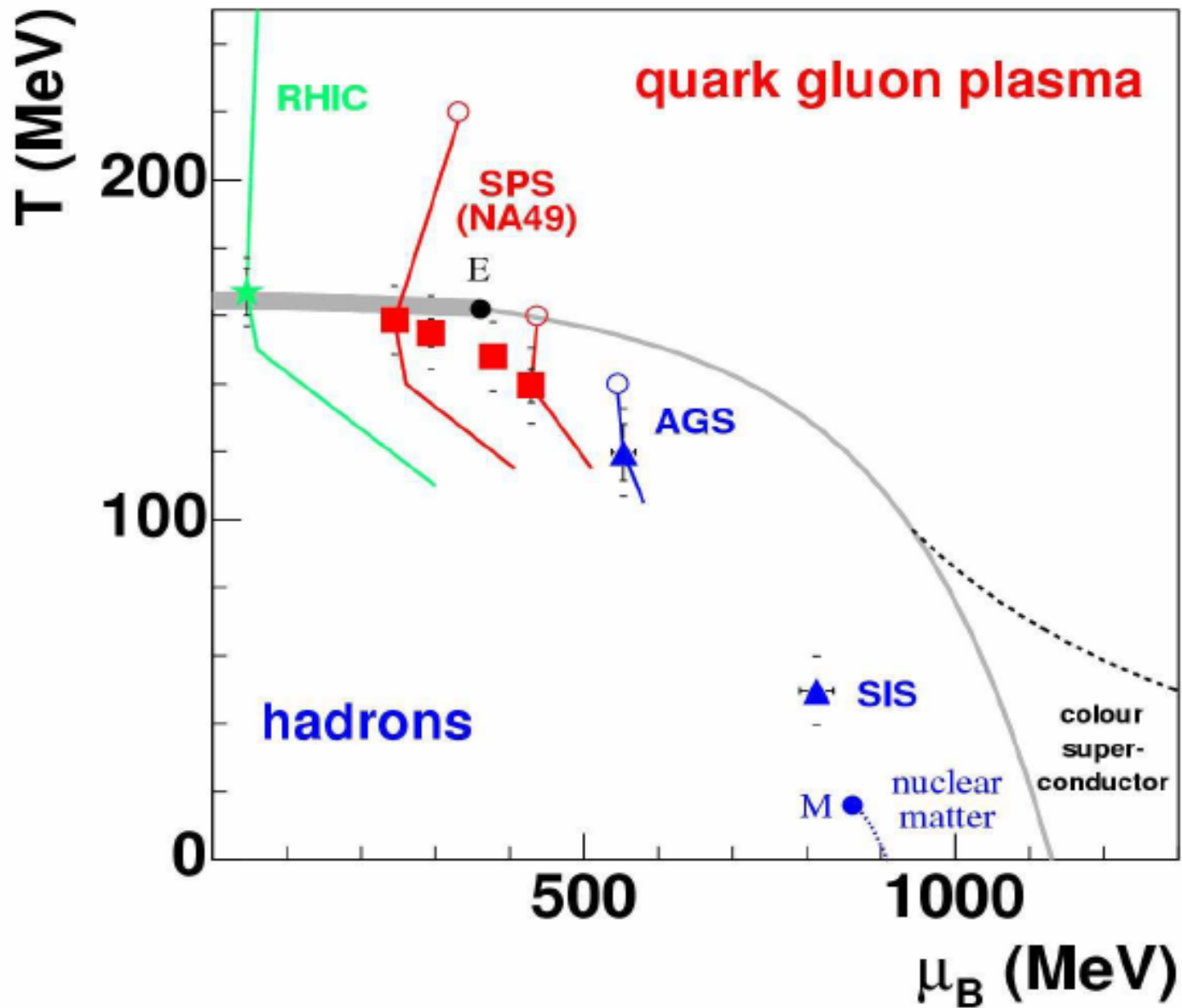
$$E, p, Q \quad T, p, Q$$

$$T, p, \mu_Q \quad E, p, \mu_Q$$

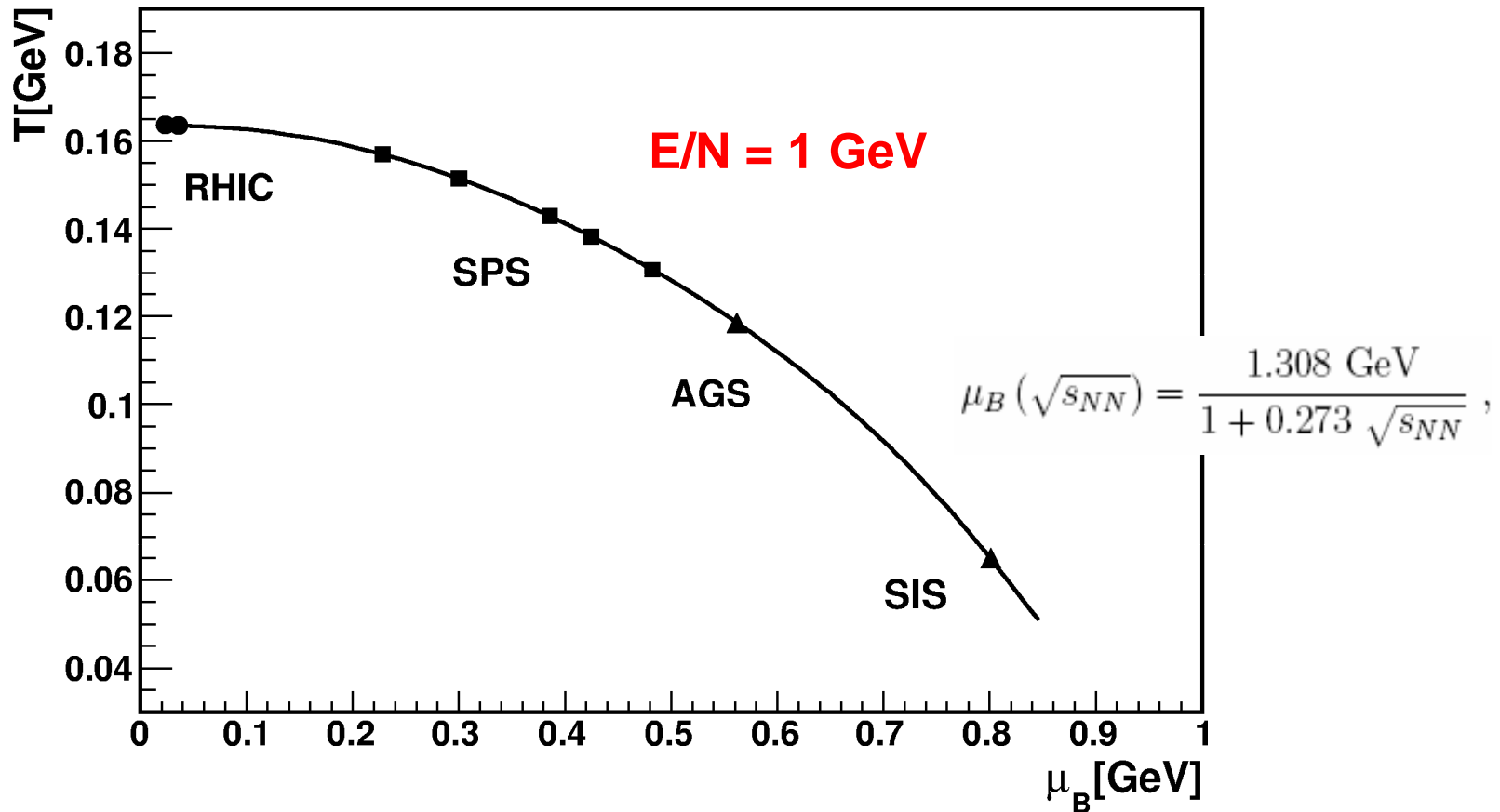
Gorenstein,
J. Phys. G
(2008)

Pressure

Ensembles

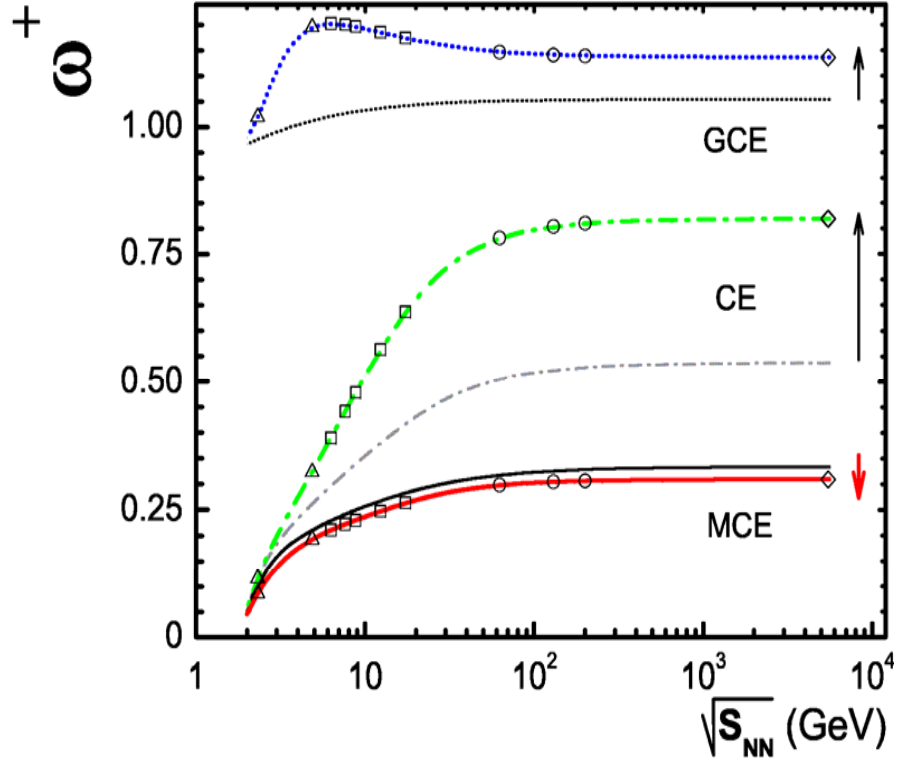
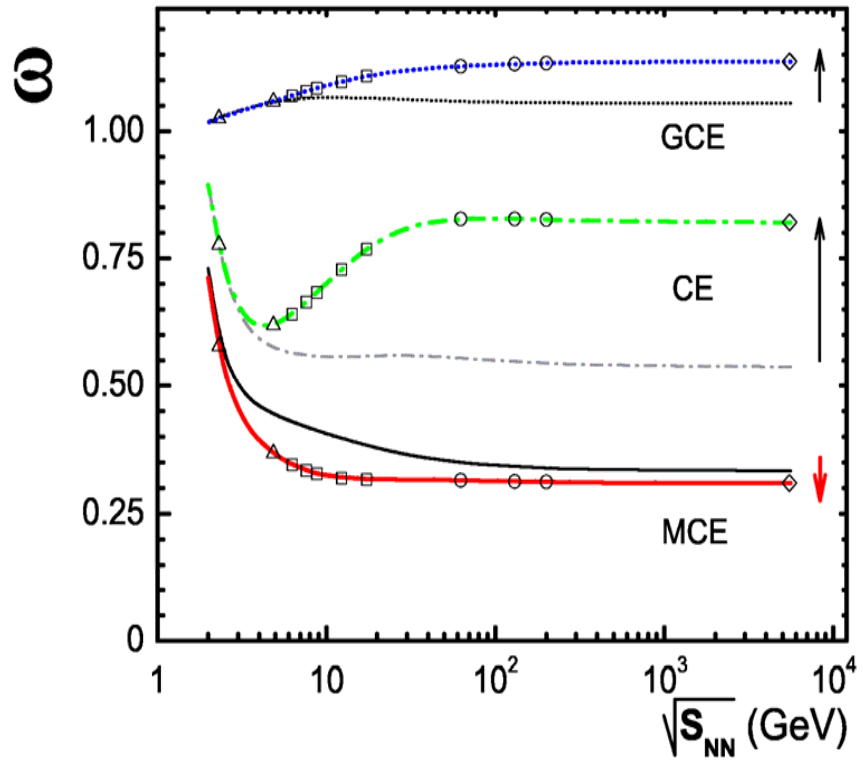


Line of the chemical freeze-out



Cleymans and Redlich, Phys. Rev. Lett. (1998)

The prediction of hadron gas model

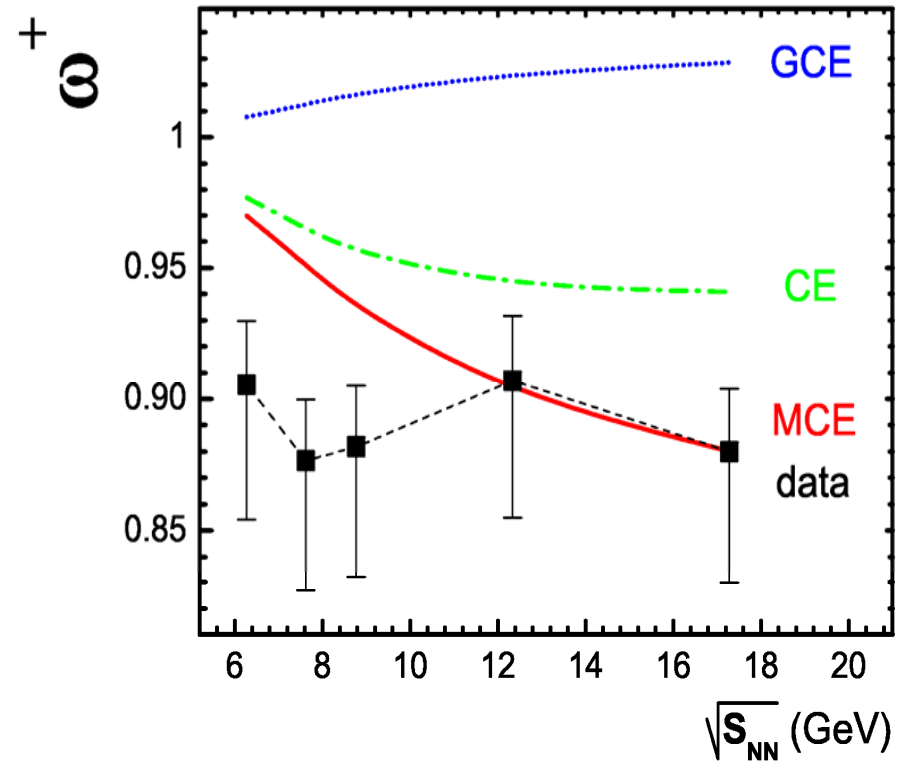
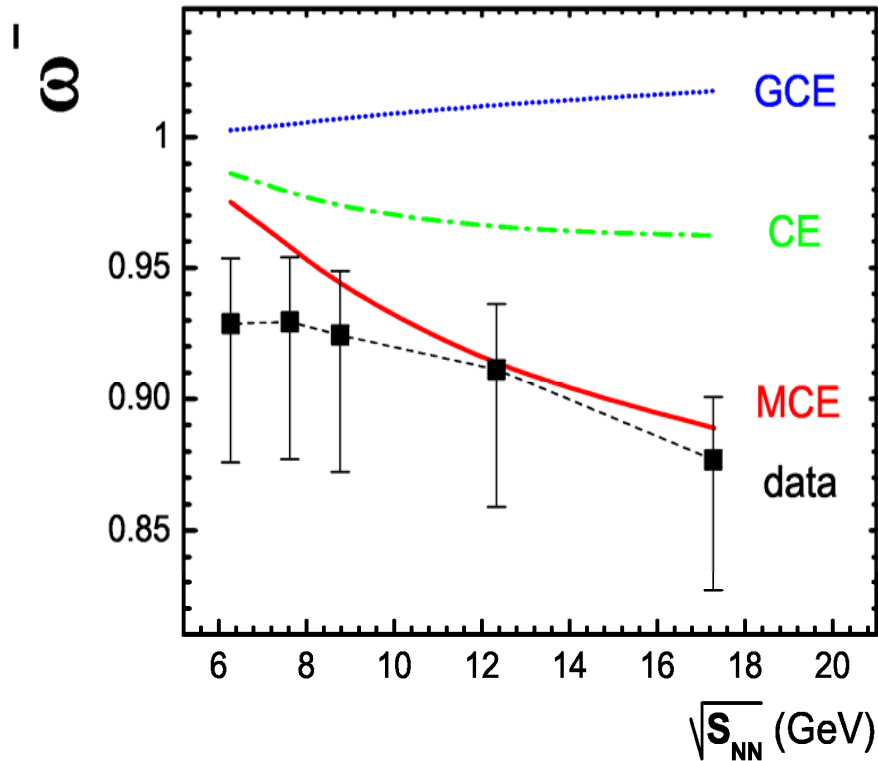


$$\omega_{4\pi}^{\pm} \equiv \frac{\langle (\Delta N_{\pm})^2 \rangle}{\langle N_{\pm} \rangle}$$

$$\omega_{acc}^{\pm} = 1 - q + q \omega_{4\pi}^{\pm}$$

**Begun, Gazdzicki, M.I.G. Hauer, Konchakovski, Lungwitz,
Phys. Rev. C (2007)**

Comparison with the NA49 data



$$q = 0.038, 0.063, 0.085, 0.131, 0.163$$

Begun, Gazdzicki, M.I.G. Hauer, Konchakovski, Lungwitz,
Phys. Rev. C (2007)

$$\vec{A} = (E, V, Q_1, \dots, Q_k)$$

Alpha-Enesmbles

$$P_\alpha(X) = \int d\vec{A} P_\alpha(\vec{A}) P_{mce}(X; \vec{A})$$

M.I.G. , Hauer, Phys. Rev. C (2008)

Problems of the statistical approach

I. “Non-Statistical” Multiplicity Distributions in

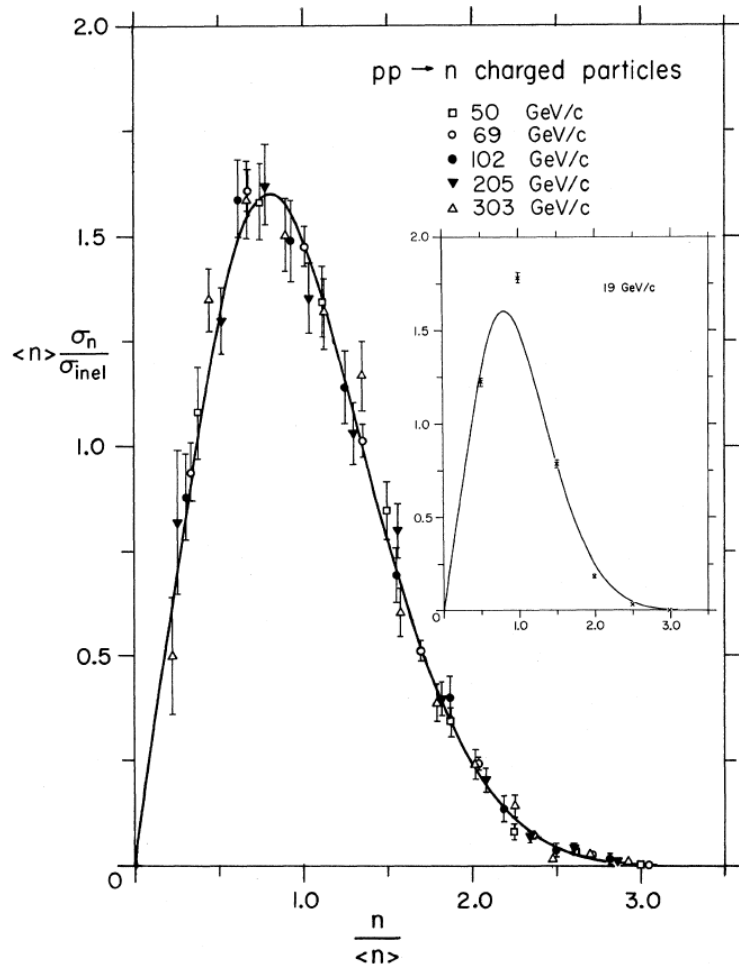
$$e^+e^-, pp, p\bar{p}$$

II. Power law at high p_T and high m

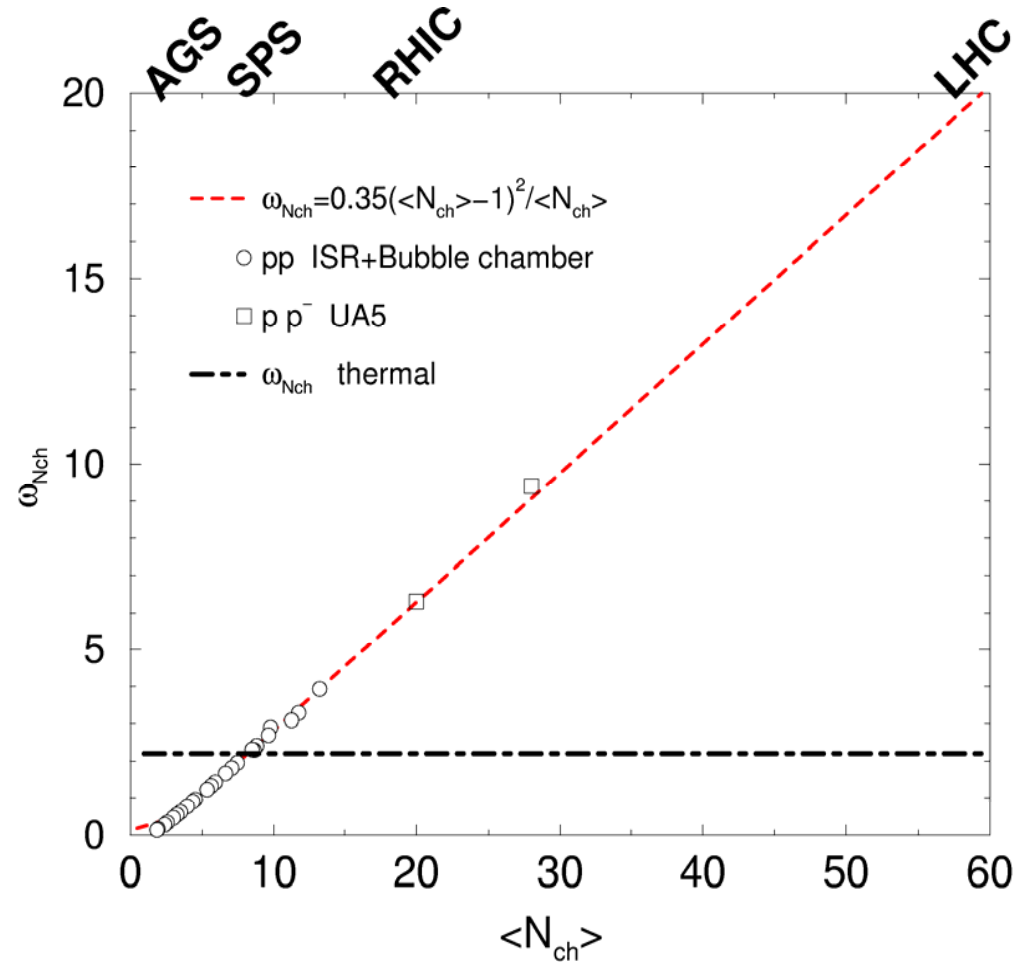
$$\frac{d^3N_i}{dp^3} \sim C p_T^{-K_p} \quad p_T \gg m_i$$

$$\langle N_i \rangle \sim C m_i^{-K_m} \quad \begin{array}{l} K_p \approx 8 \\ K_m \approx K_p - 3 \end{array}$$

KNO scaling & Large fluctuations



Slattery, Phys. Rev. Lett. (1972);



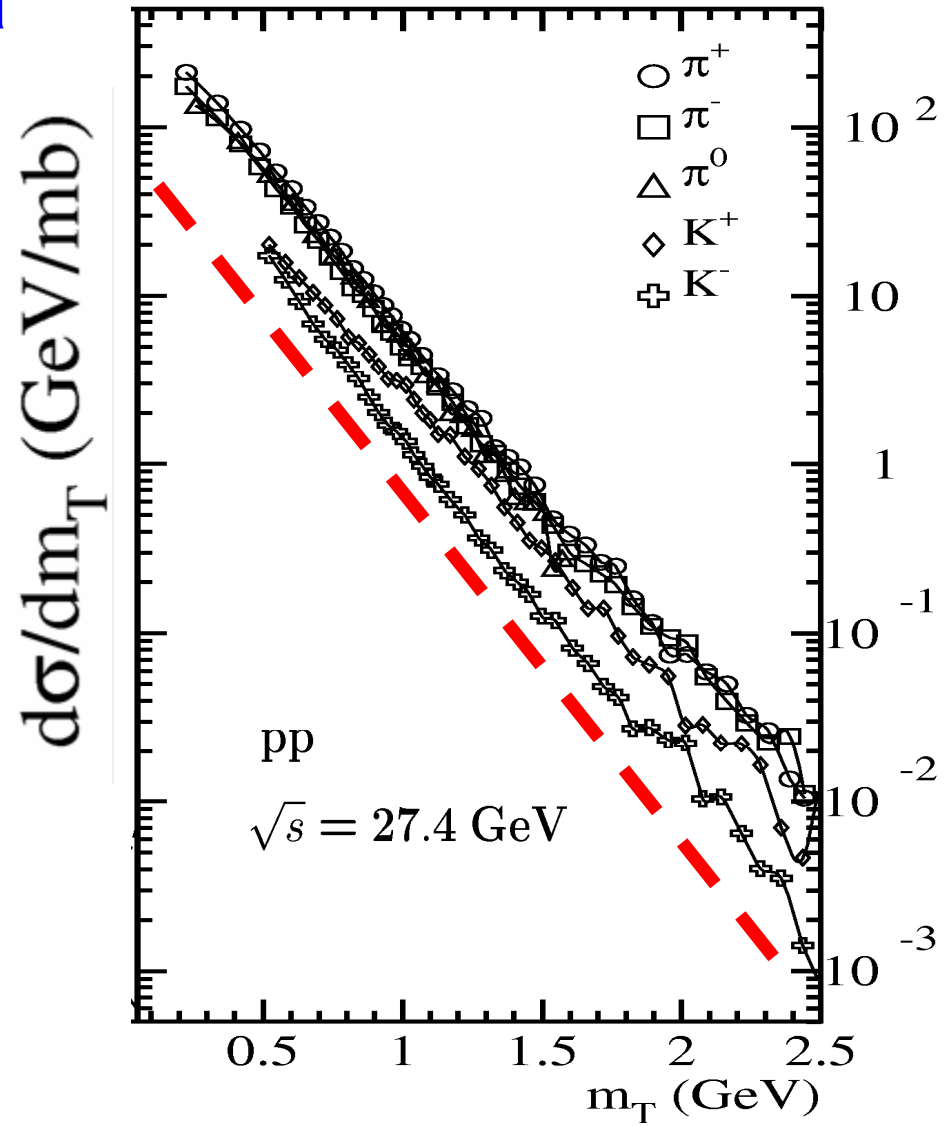
Heiselberg, Phys. Rept. (2001)

Momentum Spectra

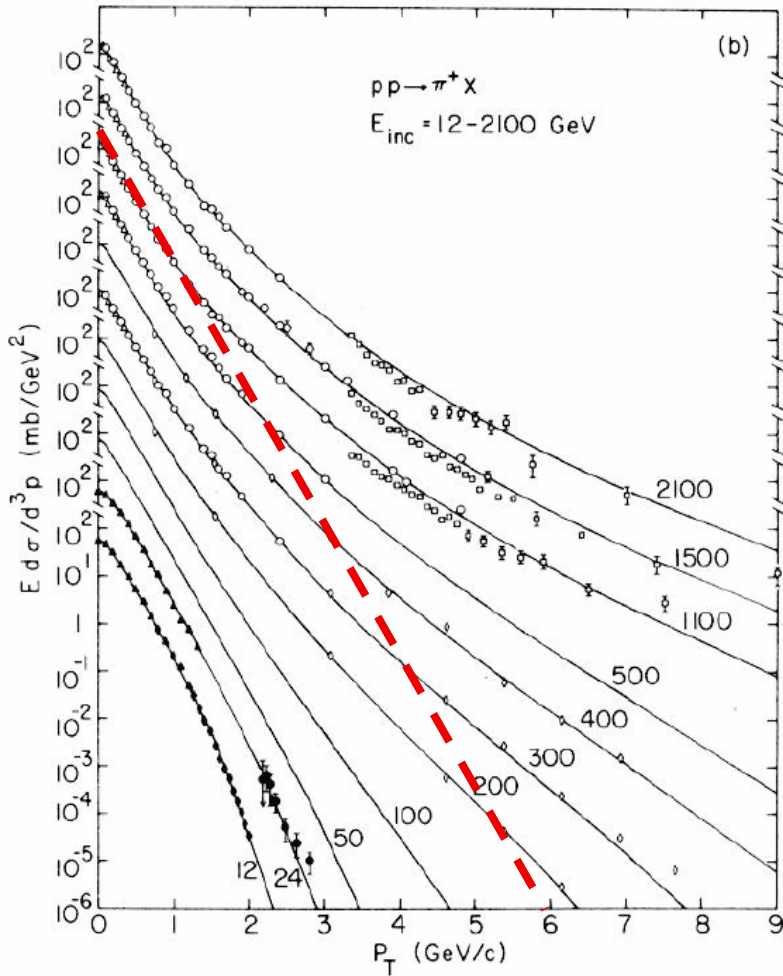
Becattini, Passaleva,
Eur. Phys. J. (2002),
data from
Aguilar-Benitez et al.,
Z. Phys. C (1990)

$$\exp\left(-\frac{m_T}{T}\right)$$

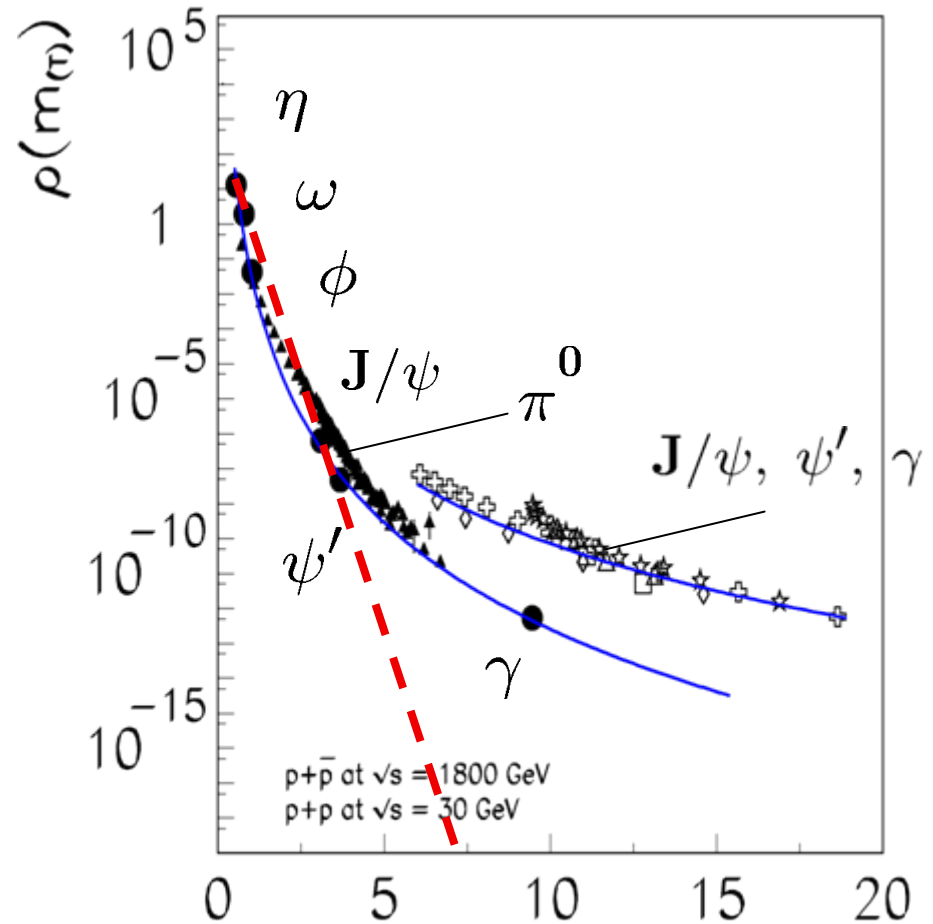
$$m_T = \sqrt{p_T^2 + m^2}$$



Power law at high p_T and high m



Beier *et al.*, Phys. Rev. D (1978)



Gazdzicki, Gorenstein,
 Phys. Lett. B (2001)

$m(m) [\text{GeV}/c^2]$

I. Multiplicity distribution

KNO scaling & Large fluctuations

Data:
$$P(N) = \frac{1}{\langle N \rangle} \Psi_{\alpha} \left(\frac{N}{\langle N \rangle} \right),$$
 Koba, Nielsen, Olesen, Nucl. Phys. B (1972)

$$\omega \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} \propto \langle N \rangle$$

Statistical Models:

$$P(N) \cong \frac{1}{\sqrt{2\pi\omega\langle N \rangle}} \exp \left[-\frac{(N - \langle N \rangle)^2}{2\omega\langle N \rangle} \right],$$

$$\omega \approx \text{const} \approx 1$$

Micro Canonical Ensemble with scaling Volume Fluctuations (MCE/sVF)

$$P_{\alpha}(\mathbf{X}; \mathbf{E}) = \int_0^{\infty} dV P_{\alpha}(V) P_{\text{mce}}(\mathbf{X}; \mathbf{E}, V)$$

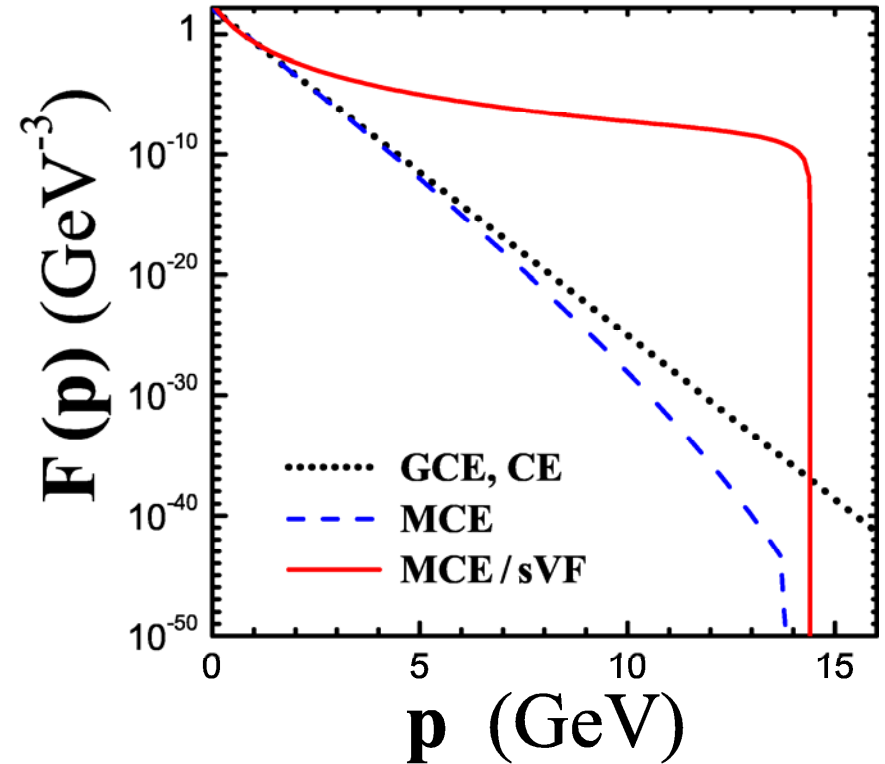
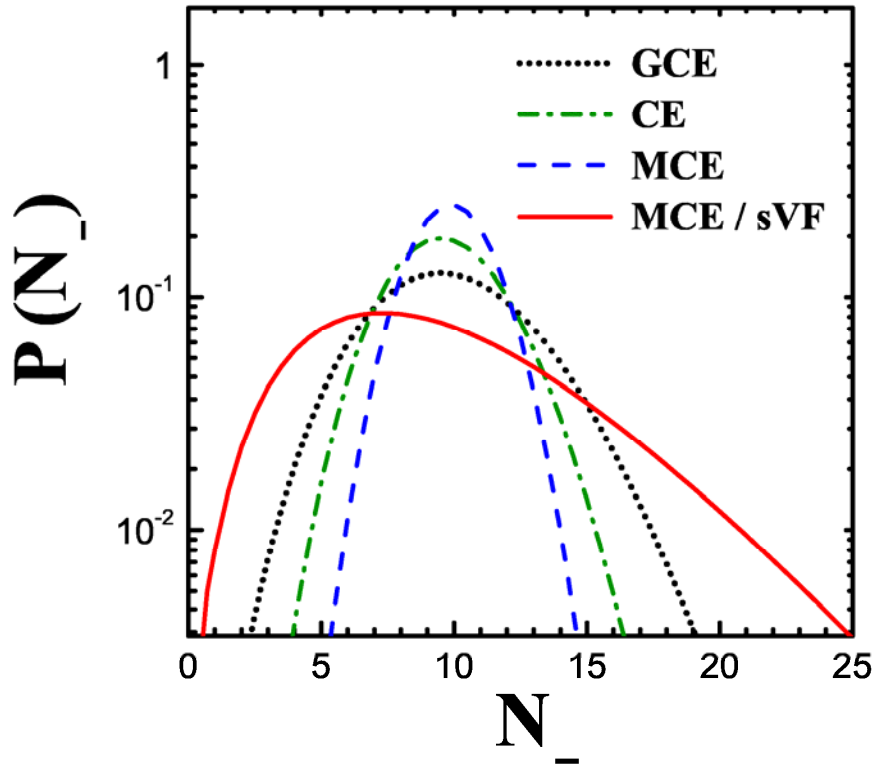
$$\mathbf{X} = \mathbf{N}, p$$

Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008)

$$P_{\alpha}(V) = \frac{1}{\bar{V}} \Phi_{\alpha}(V/\bar{V})$$

Scaling volume fluctuations selected
to fit **experimental** multiplicity **distribution**

Particle Number Distributions and Spectra

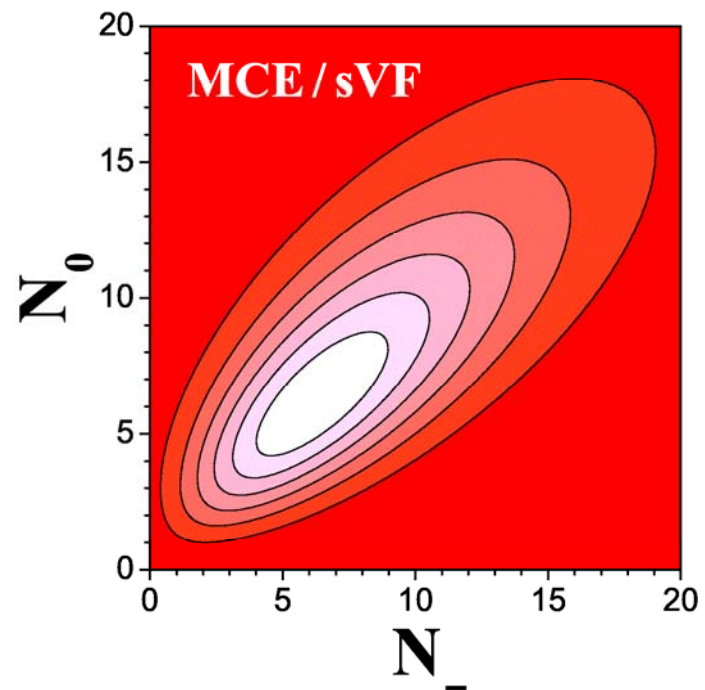
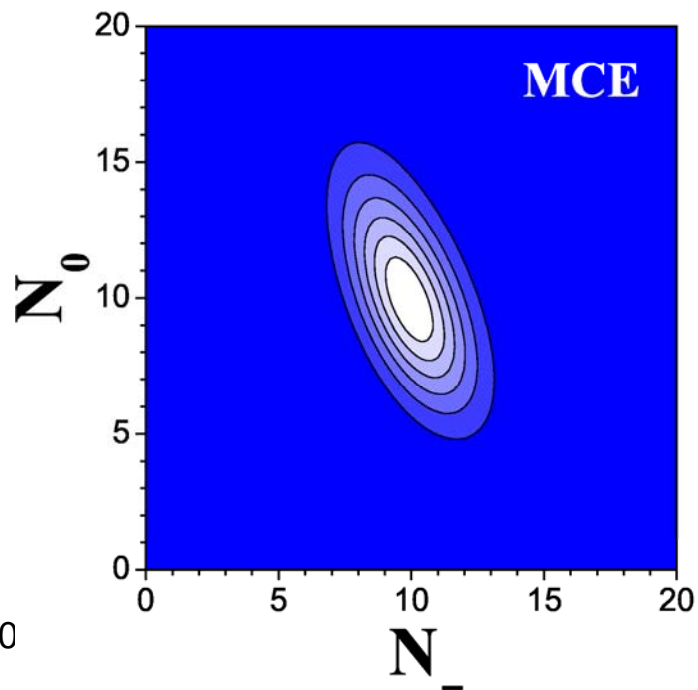
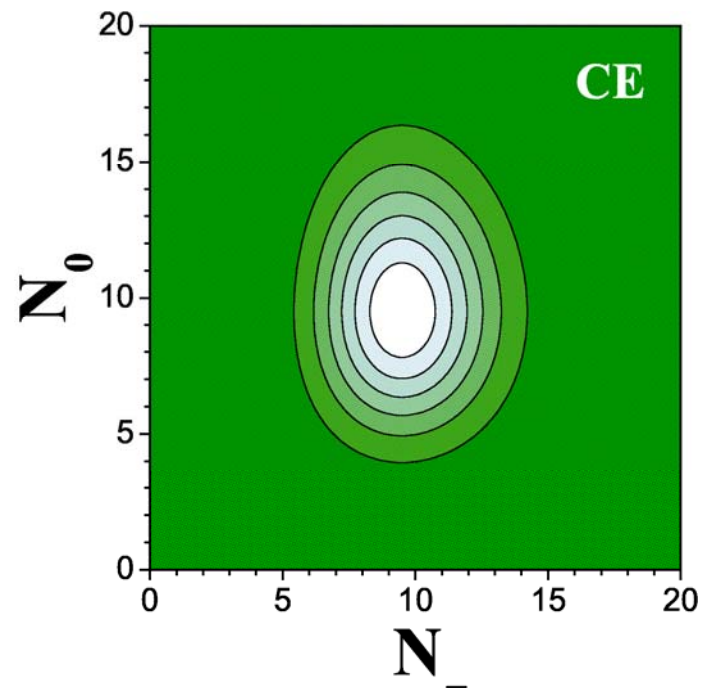
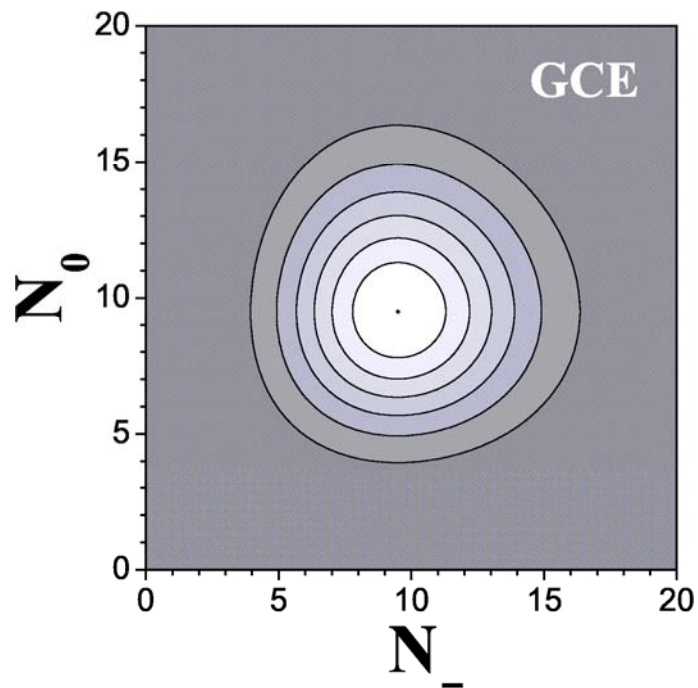


$$P_\alpha(N) = \frac{1}{\bar{N}} \Psi_\alpha(N/\bar{N})$$

$$\Psi_\alpha(y) = \frac{k^k}{3!} y^{k-1} \exp(-ky)$$

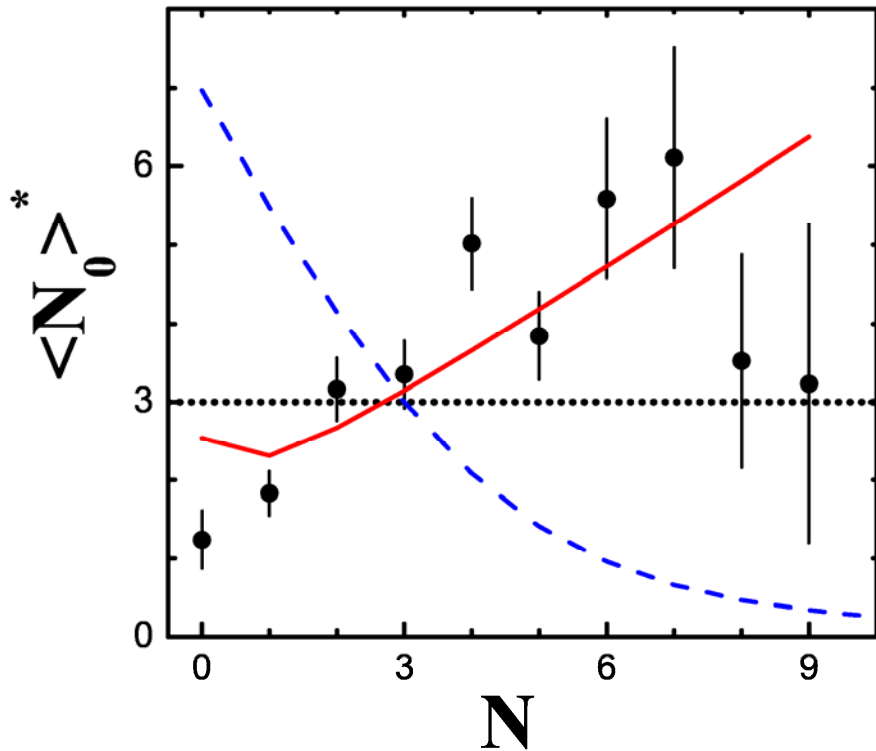
$$F_\alpha(p) \cong \frac{k^k \Gamma(k+4)}{2\Gamma(k)} T^{k+1} (p+kT)^{-k-4}$$

$$\cong 11.27 \text{ GeV}^5 (p+4T)^{-8}$$

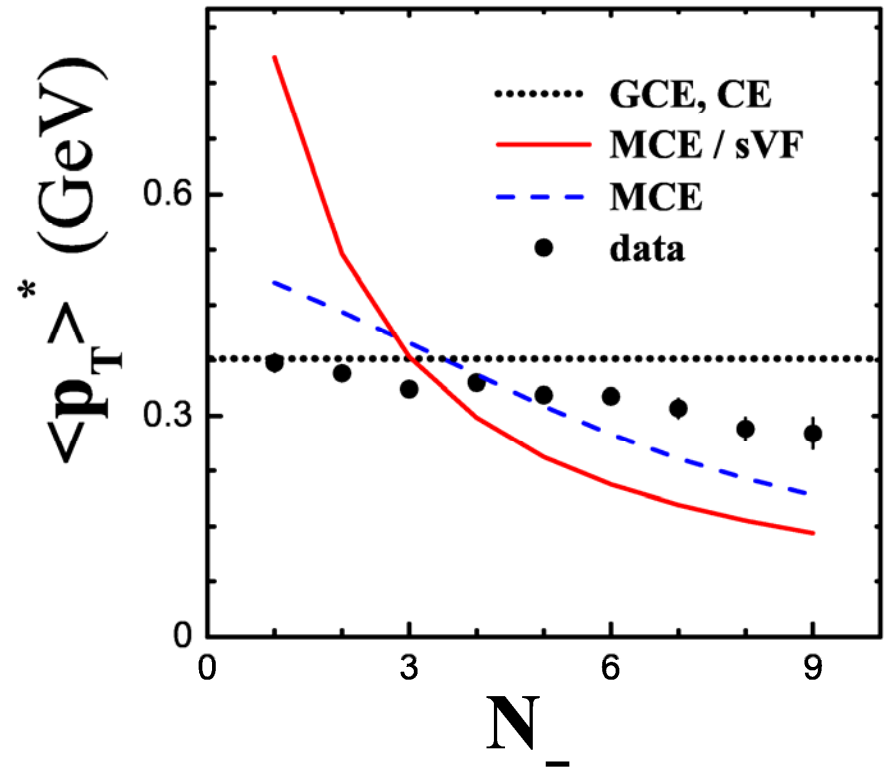


Semi-Inclusive Observables

Begun, Gazdzicki, M.I.G, Phys. Rev. (2009)



Data on $p+p$ at 205 GeV/c
Phys. Rev. D (1975) and (1977)



Summary

1. **Statistical Ensembles with Fluctuating Extensive Quantities**
2. **MCE/sVF**
 - a) **Large Particle Number Fluctuations**
 - b) **Power Law at Large Transverse Momenta**
3. **Semi-Inclusive Observables in Statistical Mechanics**