Pion and kaon femtoscopy at RHIC in hydrokinetic model

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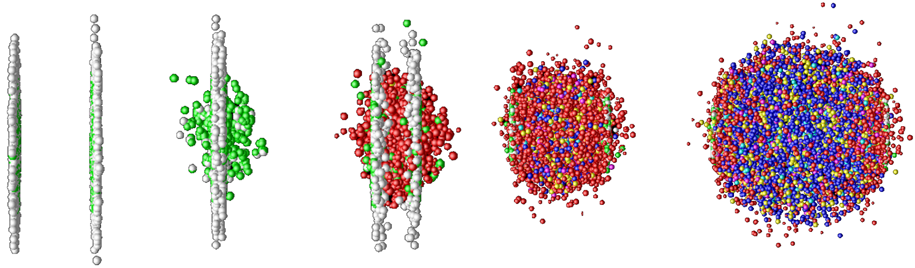
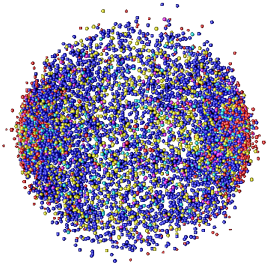
Introduction: heavy ion collision in pictures[1]



- Initial state, hard scatterings
- Thermalization
- Hydrodynamic expansion
  - Quark-gluon plasma
  - Phase transition
  - Hadron Gas
  - Chemical freeze-out
- Kinetic stage
- (kinetic) freeze-out

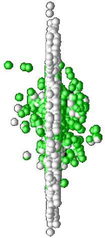
Typical size
10 fm $\propto 10^{-14}$m

Typical lifetime
10 fm/c $\propto 10^{-23}$s



[1] taken from event generator

Thermally equilibrated evolution



Initial conditions at $\tau_0 = 1$fm/c. "Effective" initial distribution, bringing average hydrodynamic results for EbE case.

- Glauber model

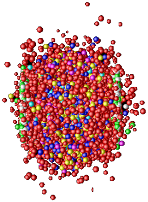
$$\varepsilon(\mathbf{b},\mathbf{r_T}) = \varepsilon_\mathbf{0}\frac{\rho(\mathbf{b},\mathbf{r_T})}{\rho_\mathbf{0}}$$

$$\rho(\mathbf{b},\mathbf{r_T}) = T(\mathbf{r_T}-\mathbf{b}/\mathbf{2})S(\mathbf{r_T}+\mathbf{b}/\mathbf{2}) + T(\mathbf{r_T}+\mathbf{b}/\mathbf{2})S(\mathbf{r_T}-\mathbf{b}/\mathbf{2})$$

- **CGC** (Color Glass condensate)

$$\varepsilon(r_T) = \varepsilon_0\frac{\rho^{3/2}(r_T)}{\rho_0^{3/2}} \quad \text{(approximate, for central collision)}$$

Rapidity profiles: $y_T = \alpha\frac{r_T}{R_T}$ (nonzero initial flow), $y_L = \eta$ (boost-inv.)
ε_0 and α are the only fitting parameters in the model.



Hydrodynamic approach

ideal fluid:

$$\partial_\nu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p\cdot g^{\mu\nu}$$

$$\partial_\mu(n_i\cdot u^\mu) = 0 \qquad \text{+equation of state } p = p(\varepsilon,\{n_i\})$$

$i = B, E, S$ in QGP phase
$i = 1\ldots N$, $N = 359$ in hadron phase (see EoS, below)

Hydrodynamics

- Bjorken coordinates :

$$\tau=(t^2-z^2)^{1/2},\quad \eta=\frac{1}{2}\ln\frac{t+z}{t-z}$$

- Conservative variables:

$$\vec{Q}=\begin{pmatrix} Q_\tau \\ Q_x \\ Q_y \\ Q_\eta \\ \{Q_{n_i}\} \end{pmatrix}=\begin{pmatrix} \gamma^2(\varepsilon+p)-p \\ \gamma^2(\varepsilon+p)v_x \\ \gamma^2(\varepsilon+p)v_y \\ \gamma^2(\varepsilon+p)v_\eta \\ \{\gamma n_i\} \end{pmatrix}$$

- Velocities in longitudinal co-moving frame:

$$v_x=v_x^{lab}\cdot\frac{\cosh y_f}{\cosh(y_f-\eta)}$$
$$v_y=v_y^{lab}\cdot\frac{\cosh y_f}{\cosh(y_f-\eta)}$$
$$v_\eta=\tanh(y_f-\eta) \tag{1}$$

- Hydrodynamic equations:

$$\underbrace{\partial_\tau\begin{pmatrix} Q_\tau \\ Q_x \\ Q_y \\ Q_\eta \\ \{Q_{n_i}\} \end{pmatrix}}_{\text{quantities}}+\underbrace{\vec{\nabla}\cdot\begin{pmatrix} Q_\tau \\ Q_x \\ Q_y \\ Q_\eta \\ \{Q_{n_i}\} \end{pmatrix}\vec{v}+\begin{pmatrix} \vec{\nabla}(p\cdot\vec{v}) \\ \partial_x p \\ \partial_y p \\ \frac{1}{\tau}\partial_\eta p \\ 0 \end{pmatrix}}_{\text{fluxes}}+\underbrace{\begin{pmatrix} (Q_\tau+p)(1+v_\eta^2)/\tau \\ Q_x/\tau \\ Q_y/\tau \\ 2Q_\eta/\tau \\ \{Q_{n_i}/\tau\} \end{pmatrix}}_{\text{sources}}=0$$

where $\vec{\nabla}=\left(\partial_x,\ \partial_y,\ \frac{1}{\tau}\partial_\eta\right)$

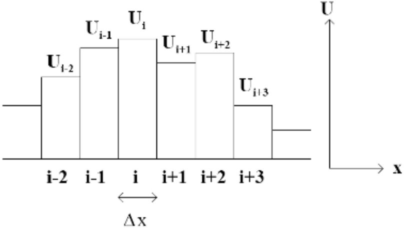
Hydrodynamics: basic method

For central A+A collisions and midrapidity: suppose longitudinal boost-ivariance and axial symmetry in transverse plane. Thus, $Q_\phi = Q_\eta = 0$, and flows $F_\phi = F_\eta = 0$.

$$\underbrace{\partial_\tau \begin{pmatrix} Q_\tau \\ Q_r \\ \{Q_{n_i}\} \end{pmatrix}}_{\text{quantities}} + \partial_r \cdot \underbrace{\begin{pmatrix} (Q_\tau + p)v_r \\ Q_r v_r + p \\ \{Q_{n_i} v_r\} \end{pmatrix}}_{\text{fluxes}} + \underbrace{\begin{pmatrix} (Q_\tau + p)(1+v_\eta^2)/\tau - (Q_\tau + p)v_r/r \\ Q_r/\tau - Q_r v_r/r \\ \{Q_{n_i}/\tau - Q_{n_i} v_r/r\} \end{pmatrix}}_{\text{sources}} = 0 \qquad (2)$$

Finite volume method:

- Divide the space into cells, Q_i in the average value of quantity inside cell



- The numerical equations:

$$Q_{ijk}^{n+1} = Q_{ijk}^{n} - \frac{\Delta t}{\Delta x_1}(F_{i+1/2,jk} + F_{i-1/2,jk}) - \frac{\Delta t}{\Delta x_2}(F_{i,j+1/2,k} + F_{i,j-1/2,k}) - \frac{\Delta t}{\Delta x_3}(F_{ij,k+1/2} + F_{ij,k-1/2})$$

F - time-averaged flow through the cell interface.

Hydrodynamics: numerical algorithm

- Flow through the cell interface depends only on the Riemann problem solution for this interface (+CFL condition)
- We use rHLLE solver for Riemann problem
- *predictor-corrector* scheme is used for the second order of accuracy in time, i.e. the numerical error is $O(dt^3)$, instead of $O(dt^2)$
- in space : in the same way, to achieve the second order scheme the *linear distributions* of quantities (conservative variables) inside cells are used.
- *Multi-dimension problem*: we use the metod, similar to operator(dimensional) splitting, but symmetric in all dimensions.
- *Grid boudaries*: we use the method of *ghost cells*, outflow boundary.
- *Vacuum treatment*: since initial grid covers both system and surrunding vacuum, we account for finite velocity of expansion into vacuum.

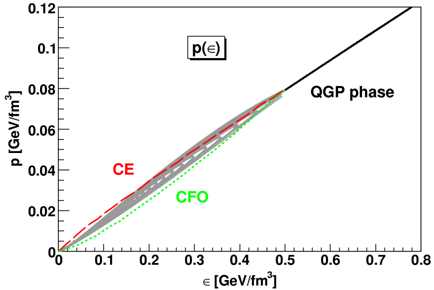
Equation of state

Equation of state, QGP, $T > T_c$ Realistic equation of state[2], consistent with lattice QCD results with crossover-type phase transition at $T_c = 175$ MeV, transforming into multicomponent hadron gas at $T = T_c$ ($\mu_B = 0$).
To account for chagre conservation in QGP phase → corrections for nonzero μ_B, μ_S [3]:

$$\frac{p(T,\mu_B,\mu_S)}{T^4} = \frac{p(T,0,0)}{T^4} + \frac{1}{2}\frac{\chi_B}{T^2}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{2}\frac{\chi_S}{T^2}\left(\frac{\mu_S}{T}\right)^2 + \frac{\chi_{BS}}{T^2}\frac{\mu_B}{T}\frac{\mu_S}{T} \quad (3)$$

Expansion coefficients χ_B, χ_S are baryon and strangs susceptibilies.

$$\frac{\mu_\alpha}{T} = const_\alpha, \quad \alpha = B, Q, S$$



Chemical freeze-out at $T_{ch} = 165 MeV$, corresponding $\mu_B = 29 MeV$, $\mu_S = 7 MeV$, $\mu_Q = -1 MeV$ and $\gamma_S = 0.935$ suppression factor, dictated by particle number ratios analysis at 200A GeV RHIC.

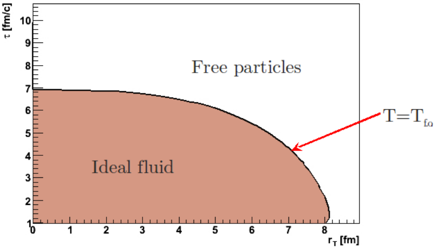
Hadron gas at $T < T_{ch}$. N=359 particle number densities are introduced, corresponding to each sort of hadrons. Yields from resonance decays are effectively included (massive resonance approximation):

$$\partial_\mu(n_i u^\mu) = -\Gamma_i n_i + \sum_j b_{ij}\Gamma_j n_j$$

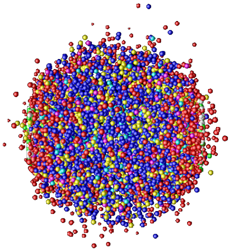
Grey points: differen chemical compositions

[2] M. Laine, Y. Schroder Phys. Rev. D73 (2006) 085009.
[3] F. Karsch, PoS CPOD07:026, 2007.

Final stage of evolution



Connecting hydrodynamic and kinetic(final) stage: Cooper-Frye prescription

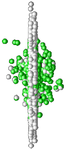


Final stage (weakly interacting system)

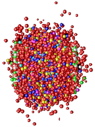
- UrQMD (afterburner)
  C. Nonaka, S.A. Bass, Phys. Rev. C75 (2007) 014902
- JAM (afterburner)
  T. Hirano, M. Gyulassy, Nucl. Phys. A769 (2006) 71-94
- THERMINATOR
  W. Florkowski, W. Broniowski, M. Chojnacki, A. Kisiel, Acta Phys.Polon.B40 (2009), 1093-1098

Hydro-kinetic model

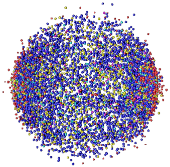
Goal: to connect hydrodynamic and final stages in a more natural way (not Cooper-Frye prescription), and account for the deviations from local equilibrium at hydrodynamic stage (remember viscosity?).



Initial stage: CGC, Glauber models



QGP phase: ideal hydrodynamics, “Lattice QCD” EoS



Hadron phase (late hydrodynamic+kinetic stages): **Hydro-kinetic approach**
Yu.M. Sinyukov, S.V. Akkelin, Y. Hama, Phys.Rev.Lett.89:052301,2002

Hydro-kinetic approach to particle emission

- is based on relaxation time approximation for emission function of relativistic finite expanding system
- provides evaluation of escape probabilities and deviations of distribution functions from local equilibrium
  Zero approximation is ideal hydro.

Complete algorithm:

- solution of equations of ideal hydro
- calculation of non-equilibrium DF and emission function in first approximation
- solution of equations for ideal hydro with non-zero left-hand-side that accounts for conservation laws for non-equilibrium process of the system which radiated free particles during expansion
- Calculation of “exact” DF and emission function
- Evaluation of spectra and correlations

Hydrokinetic approach to particle emission

Particle liberation from hydrodynamically expanding system is described by the approximate method inspired by the integral form of Boltzmann equation [4]:

$$f_i(t,\vec{x},p) = f_i(\bar{x}_{t\to t_0},p)\mathscr{P}_{t_0\to t}(\bar{x}_{t\to t_0},p) + \int\limits_{t_0}^{t} \underbrace{G_i((\bar{x}_{t\to s},p))\mathscr{P}_{s\to t}(\bar{x}_{t\to s},p)}_{S(\bar{x}_{t\to s},p)} ds, \qquad \bar{x}_{t\to s} = (s, \vec{x} + \frac{\vec{p}}{p^0}(s-t))$$

where $\frac{p^\mu}{p^0}\frac{\partial f_i(x,p)}{\partial x^\mu} = G_i(x,p) - L_i(x,p)$, $L_i(x,p) = R_i(x,p)f_i(x,p)$ and $\mathscr{P}_{t\to t'}(x,p) = \exp\left(-\int\limits_{t}^{t'} d\bar{t}R_i(\bar{x}_t,p)\right)$

Relaxation time approximation for collision terms (if i=stable particle):

$$R_i(x,p) \approx R_i^{l.eq.}(x,p) = \text{collision rate}, \quad \text{and} \quad G_i \approx R_i^{l.eq.}(x,p)f_i^{l.eq.}(x,p) + G_i^{\text{decay}}(x,p)$$

$$\Downarrow$$

$$\frac{p^\mu}{p_0}\frac{\partial f_i(x,p)}{\partial x^\mu} = -\frac{f_i(x,p) - f_i^{\text{l.eq.}}(x,p)}{\tau_{\text{rel}}(x,p)} + G_i^{\text{decay}}(x,p).$$

First approximation (ideal hydro!):

$$\partial_\nu T_i^{\nu\mu}[f_i^{\text{l eq}}] = 0, \quad \partial_\nu n^\nu[f_i^{\text{l eq}}] = 0$$

The "relaxation time" $\tau_{rel} = 1/R_i^{l.eq.}$ grows with time!

For i-th coomponent of hadron gas, in Bjorken coordinates:

Emission function

$$S_i(\lambda,\theta,r_T,p) = \left[f_i^{l.eq.}(\lambda,\theta,r_T,p)\tilde{R}_i(\lambda,\theta,r_T,p) + \tilde{G}_i^{decay}(\lambda,\theta,r_T,p)\right]\exp\left(-\int\limits_{\lambda}^{\infty}\tilde{R}_i(s,\theta^{(s)}(\lambda),r_T^{(s)}(\lambda),p)ds\right)$$

[4] for the details, see Yu.M. Sinyukov, S.V. Akkelin, Y. Hama, Phys.Rev.Lett.89:052301,2002

$$p_i^0 G_i^{decay}(x,p_i) = \sum_j \sum_k \int \frac{d^3 p_j}{p_j^0} \int \frac{d^3 p_k}{p_k^0} \Gamma_{j\to ik} f_j(x,p_j) \frac{m_j}{F_{j\to ik}} \delta^{(4)}(p_j - p_k - p_i)$$

Collision rate (inverse relaxation time) for i-th sort of hadrons:

$$\frac{1}{\tau_{i,\mathrm{rel}}^{\mathrm{id}*}(x,p)} = R_i^{\mathrm{id}}(x,p) = \int \frac{d^3k}{(2\pi)^3} \exp\left(-\frac{E_k - \mu_{id}(x)}{T_{id}(x)}\right) \sigma_i(s) \frac{\sqrt{s(s-4m^2)}}{2E_p E_k}.$$

where $E_p = \sqrt{\mathbf{p}^2 + m^2}$, $E_k = \sqrt{\mathbf{k}^2 + m^2}$, $s = (p+k)^2$ is squared pair energy in CMS, $\sigma(s)$ - cross-section, calculated in a way similar to URQMD. Observable quantity: particle spectrum,

$$\frac{d^3 N_i}{d^3 p} = n_i(p) = \int_{t\to\infty} d^3x \; f_i(t,x,p)$$

Kinetics: inverse relaxation time (collision rate)

collision rate (inverse relaxation time):

$$\frac{1}{\tau_{\rm rel}^{\rm id*}(x,p)} = R^{\rm id}(x,p) = \int \frac{d^3k}{(2\pi)^3} \exp\left(-\frac{E_k - \mu_{id}(x)}{T_{id}(x)}\right) \sigma(s) \frac{\sqrt{s(s-4m^2)}}{2E_pE_k}.$$

where $E_p = \sqrt{\mathbf{p}^2 + m^2}$, $E_k = \sqrt{\mathbf{k}^2 + m^2}$, $s = (p+k)^2$ is squared pair energy in CMS, $\sigma(s)$ - cross-section, calculated in a way similar to URQMD.:

- meson-meson, meson-baryon:

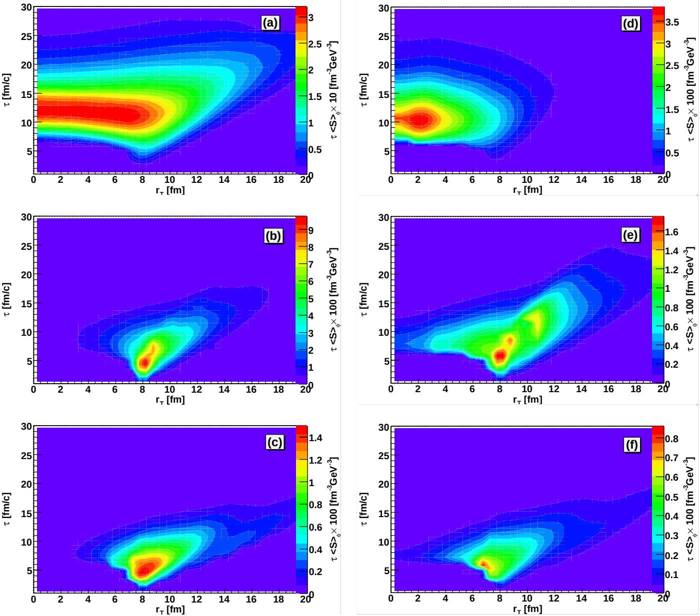
$$\sigma_{tot}^{MB}(\sqrt{s}) = \sum_{R=\Delta,N^*} \langle j_B, m_B, j_M, m_M \| J_R, M_R \rangle \frac{2S_R+1}{(2S_B+1)(2S_M+1)} \times \frac{\pi}{p_{cm}^2} \frac{\Gamma_{R\to MB}\Gamma_{tot}}{(M_R-\sqrt{s})^2 + \Gamma_{tot}^2/4} \quad ,$$

  +5 mbarn for elastic meson-meson scattering
- $p-p$, $p-n$, $p-\bar{p}$, etc. $\to \to$ tables
- other: additive quark model:

$$\sigma_{\rm total} = 40 \left(\frac{2}{3}\right)^{m_1+m_2} \left(1 - 0.4\frac{s_1}{3-m_1}\right) \left(1 - 0.4\frac{s_2}{3-m_2}\right) [\rm mb] \quad ,$$

  $m_i = 1(0)$ for meson (baryon), s_i - number of strange quarks in particle i.

Hydro-kinetic approach: Emission functions

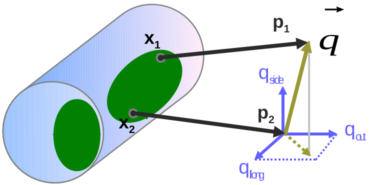


Emission function = the probability for particle to be emitted from space-time point with given momentum.

Emission functions for π^- (left) and K^- (right) at RHIC 200A GeV central collisions.
(a,d) $p_T = 0.2$ GeV
(b) $p_T = 0.85$ GeV
(e) $p_T = 0.7$ GeV
(c,f) $p_T = 1.2$ GeV

Unlike “hybird” models, this one catches the emission from dense part of system, where transport approach is not valid.

HBT(interferometry) measurements



$$\vec{q} = \vec{p}_2 - \vec{p}_1$$

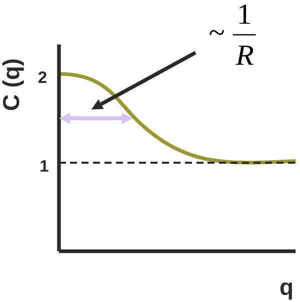
$$\vec{k} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$$

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\text{real event pairs}}{\text{mixed event pairs}}$$

Gaussian approximation of CFs
($q \to 0$):

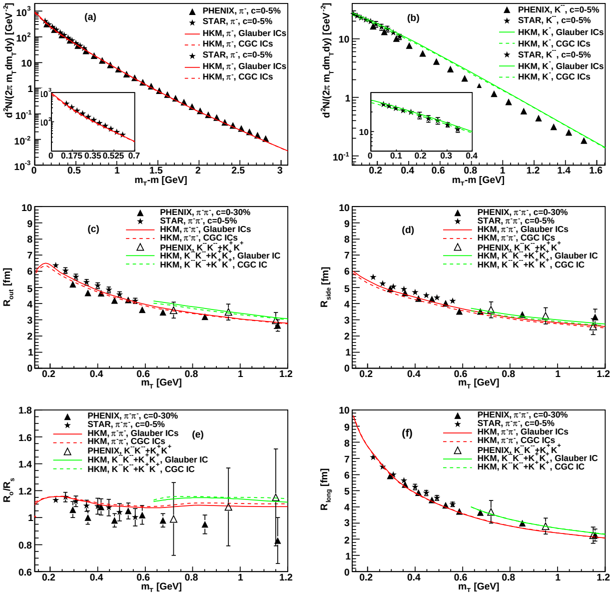
$$C(\vec{k}, \vec{q}) = 1 + \lambda(k) e^{-q_{out}^2 R_{out}^2 - q_{side}^2 R_{side}^2 - q_{long}^2 R_{long}^2}$$

$R_{out}, R_{side}, R_{long}$ (HBT radii)
correspond to homogeneity lengths,
which reflect the space-time scales of
emission process



However, hybrid models usually cannot reproduce R_i measured in experiment

Results: spectra+interferometry(HBT) radii for 200A GeV RHIC

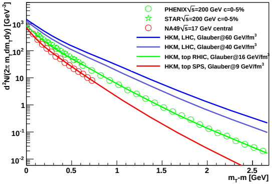


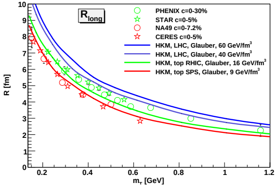
The transverse momentum spectra of negative pions and negative kaons in HKM model; the interferometry radii and R_{out}/R_{side} ratio for $\pi^-\pi^-$ pairs and mixture of K^-K^- and K^+K^+ pairs. The experimental data for 200A GeV collisions are taken from the STAR and PHENIX Collaborations.

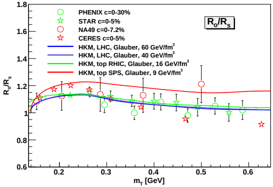
- Glauber IC:
  $\epsilon_0 = 16.5$ GeV/fm^3
  $<\epsilon>= 11.7$ GeV/fm^3
  $<v_T>= 0.224$
- CGC IC:
  $\epsilon_0 = 19.5$ GeV/fm^3
  $<\epsilon>= 13.2$ GeV/fm^3
  $<v_T>= 0.208$

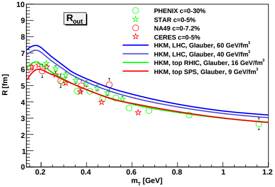
Bigger $<v_T>$ accumulates viscosity effects, EbE, etc

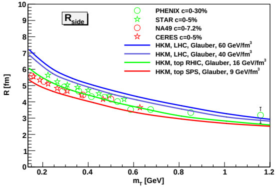
Results: top SPS + top RHIC + LHC predictions











Model parameters:

	ϵ_0	α
top SPS	9	0.194
RHIC	16.5	0.28
LHC(1)	40	0.28
LHC(2)	60	0.28

Initial energy density estimate for LHC is taken from CGC model (T. Lappi).

Conclusions

- The hydrokinetic approach to A+A collisions is developed. It allows one to describe the continuous particle emission from a hot and dense finite system, expanding hydrodynamically into vacuum, in the way which is consistent with Boltzmann equations.
- The model is extended to include realistic features of heavy ion collisions (Lattice QCD+HG EoS, resonance decays). Short-lived resonance decay contributions are incorporated in EoS and are calculated dynamically.
- An agreement with experimental data for Au+Au collisions at $\sqrt{s} = 200 AGeV$ RHIC : pion and kaon m_T spectra, pion and kaon HBT radii in hydro-kinetic model is achieved.
- Model is applied for SPS data description and LHC predictions.

Tasks in progress:

- Connection with UrQMD.
- Model extension to 2+1 case (noncentral collisions, v_2...).
- Next approximation for distribution function, emission function, etc

Thank you!

Extra slides

Hydrokinetics: relaxation time approximation for emission function

For 1-component system:

$$\frac{p^\mu}{p^0}\frac{\partial f_i(x,p)}{\partial x^\mu} = G_i(x,p) - L_i(x,p)$$

$$f(t,\vec{x},p) = f(\bar{x}_{t\to t_0},p)\mathscr{P}_{t_0\to t}(\bar{x}_{t\to t_0},p) + \int\limits_{t_0}^{t} \underbrace{G((\bar{x}_{t\to s},p))\mathscr{P}_{s\to t}(\bar{x}_{t\to s},p)}_{S(\bar{x}_{t\to s},p)} ds$$

$$\frac{d^3N}{d^3p}(t) = n(t,p) = \int d^3x f(t,x,p) \qquad \bar{x}_{t\to s} = (s, \vec{x} - \frac{\vec{p}}{p^0}(t-s))$$

$$L_i(x,p) = R_i(x,p) f_i(x,p)$$

$$R(x,p) \approx R_{l.eq.}(x,p), G \approx R_{l.eq.}(x,p) f_{l.eq.}(x,p). \quad (4)$$

$$\frac{p^\mu}{p_0}\frac{\partial f(x,p)}{\partial x^\mu} = -\frac{f(x,p) - f^{\mathrm{l\ eq}}(x,p)}{\tau_{\mathrm{rel}}(x,p)}.$$

Approximate solution:

$$f = f^{\mathrm{l\ eq}}(x,p) + g(x,p), \quad \text{äå}$$

$$g(x,p) = -\int\limits_{t_0}^{t} \frac{df^{\mathrm{l\ eq}}(t', \mathbf{r} - \frac{\mathbf{p}}{p_0}(t-t'), p)}{dt'} \exp\left\{ -\int\limits_{t'}^{t} \frac{1}{\tau_{\mathrm{rel}}(s, \mathbf{r} - \frac{\mathbf{p}}{p_0}(t-s), p)} ds \right\} dt'.$$

Hydrokinetics: general formalism

$$\partial_\nu T^{\nu\beta}[f^{\text{l eq}}] = G^\beta[g], \quad (5)$$

where

$$G^\beta[g] = -\partial_\nu T^{\nu\beta}[g]. \quad (6)$$

for particle number:

$$\partial_\nu n^\nu[f^{\text{l eq}}] = S[g], \quad (7)$$

where

$$S[g] = -\partial_\nu n^\nu[g]. \quad (8)$$

To find the approximate solution of BE,
1. Solve ideal hydro equations:

$$\partial_\nu T^{\nu\mu}[f^{\text{l eq}}] = 0, \quad (9)$$

$$\partial_\nu n^\nu[f^{\text{l eq}}] = 0, \quad (10)$$

2. Use the values obtained to calculate the deviations from equilibrium $g(x,p)$, and use them to solve (5,7) in the following approximation:

$$\partial_\nu T^{\nu\beta}[f^{\text{l eq}}(T,u_\mu,\mu)] = G^\beta[T_{\text{id}},u_\mu^{\text{id}},\mu_{\text{id}},\tau_{\text{rel}}^{\text{id}}], \quad (11)$$

$$\partial_\nu n^\nu[f^{\text{l eq}}(T,u_\mu,\mu)] = S[T_{\text{id}},u_\mu^{\text{id}},\mu_{\text{id}},\tau_{\text{rel}}^{\text{id}}], \quad (12)$$