Photon and hadron observables from a 3+1 dimensional hydro model

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- A 3+1 dimensional model
- Hadron spectra, flow and HBT
- Photon observables

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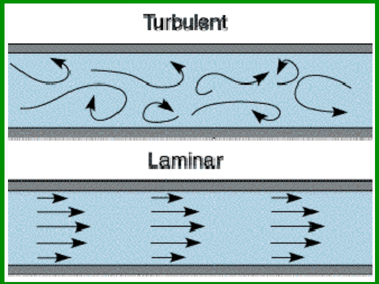
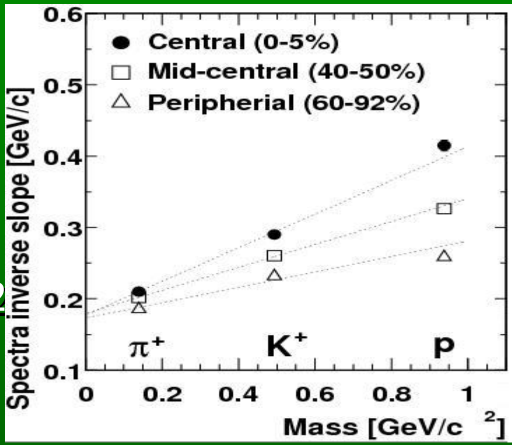
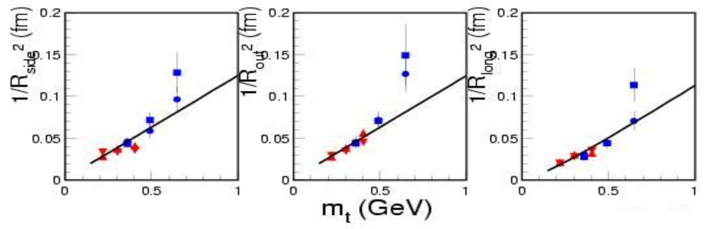
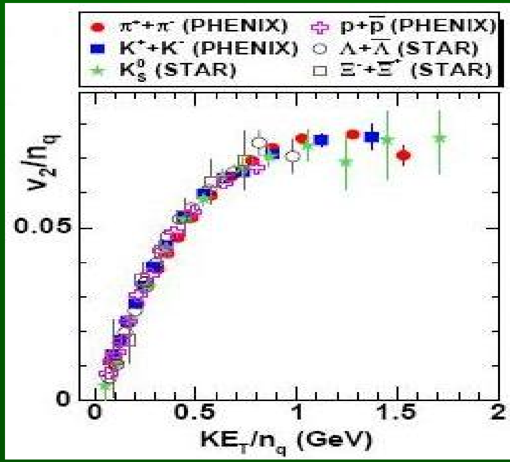
Hydrodynamic scaling predictions

- Hydro predicts scaling (also viscous)
- What does a scaling mean?
  - For example Reynolds number $\rho v r/\eta$
  - Only a combination of parameters matters
- Collective, thermal behavior → Loss of information
- Spectra slopes: $N_1 \sim e^{-\frac{m_t}{T_{\text{eff}}}};\ T_{\text{eff}} = T_0 + m u_t^2$
- Elliptic flow: $v_2 = \dfrac{I_1(w)}{I_0(w)};\ w \sim KE_T$
- HBT radii: $R_{\text{side}}^2 \approx R_{\text{long}}^2 \approx R_{\text{out}}^2 \sim \dfrac{1}{m_t}$

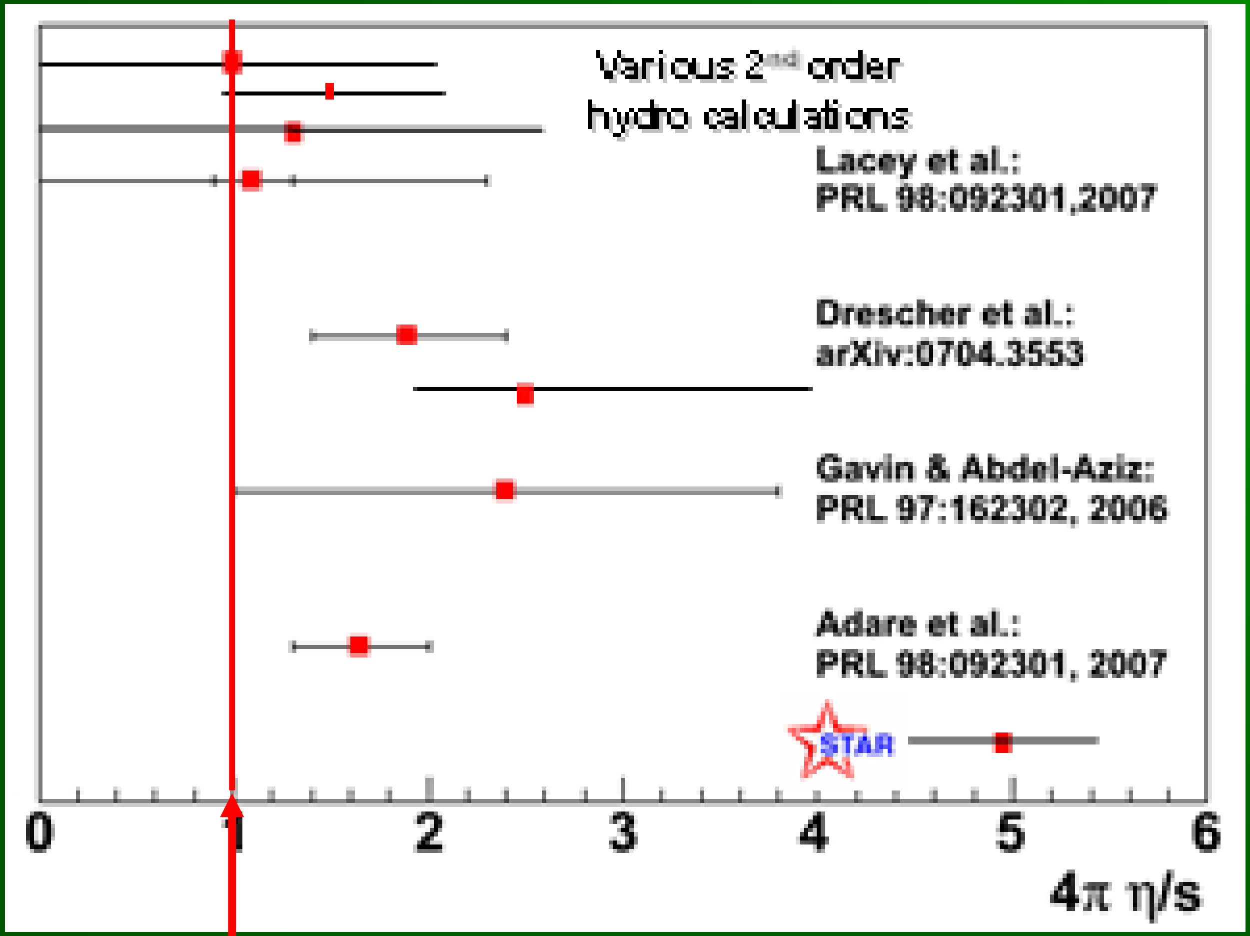








Perfect hydro picture

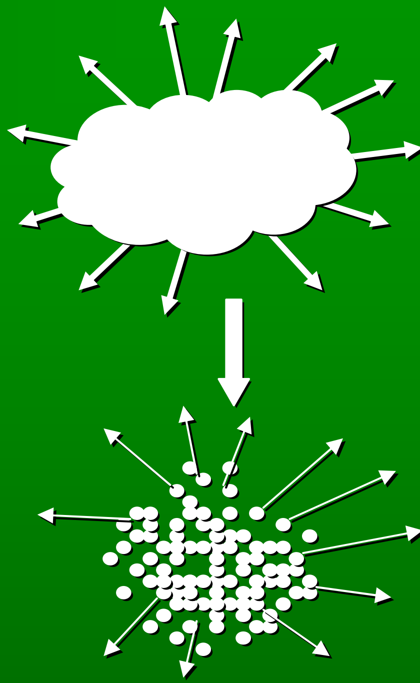


- No data point even near the kinematic viscosity of ^{4}He $(10/4\pi)$
- Close to AdS/CFT minimum, $(1/4\pi)$

Little vocabulary of hydrodynamics

- Exact/parametric solution
  - Solution of hydro equations analytically, without approximation
  - Usually has free parameters
- Hydro inspired parameterization
  - Distribution determined at freeze-out only, their time dependence is not considered
- Numerical solution
  - Solution of hydro equations numerically
  - Start from arbitrary initial state

How analytic hydro works



- Take hydro equations and EoS
- Find a solution
  - Will contain parameters (like Friedmann, Schwarzschild etc.)
  - Will use a possible set of initial conditions
- Use a freeze-out condition
  - Eg fixed proper time or fixed temperature
  - Generally a hyper-surface
- Calculate the hadron source function
- Integrate over time to calculate photon spectra
- Hydrodynamics: Initial conditions ⊗ dynamical equations ⊗ freeze-out conditions

Some known relativistic solutions

Solution	Basic prop's	EoS	Observables
Csörgő, Nagy, Csanád Phys.Lett.B 663:306-311, 2008 Phys.Rev.C77:024908,2008	Ellipsoidal, 1D accelerating	$\varepsilon - B = \kappa(p+B)$	dn/dy, ε
Landau Izv. Acad. Nauk SSSR 81 (1953) 51	Cylindr., 1D, accelerating	$\varepsilon = \kappa nT$	none
Hwa-Björken R.C. Hwa, PRD10, 2260,1974 J.D. Bjorken, PRD27, 40(1983)	Cylindr., 1D, non-accelerating	$\varepsilon = \kappa nT$	dn/dy, ε
Bialas et al. A. Bialas, R. A. Janik, and R. B. Peschanski, Phys. Rev. C76, 054901 (2007).	1D, betweend Landau and Hwa-Björken	$\varepsilon = \kappa nT$	dn/dy
Csörgő, Csernai, Hama, Kodama Heavy Ion Phys. A 21, 73 (2004))	Ellipsoidal, 3D, non-accelerating	$\varepsilon = \kappa nT$	This work does the calculation

Where we are

- Revival of interest, new solutions
  - Sinyukov, Karpenko, nucl-th/0505041
  - Pratt, nucl-th/0612010
  - Bialas et al.: Phys.Rev.C76:054901,2007
  - Borsch, Zhdanov: SIGMA 3:116,2007
  - Nagy et al.: J.Phys.G35:104128,2008 and arXiv/0909.4285
  - Liao et al.: arXiv/09092284 and Phys.Rev.C80:034904,2009
  - Mizoguchi et al.: Eur.Phys.J.A40:99-108,2009
  - Beuf et al.:Phys.Rev.C78:064909,2008 (dS/dy as well!)
- Need for solutions that are:
  - accelerating + relativistic+ 3 dimensional
  - explicit + simple + compatible with the data
- Need to calculate observables!

The solution we investigate

- Density, temperature, pressure
  - ν(s) arbitrary, but realistic to choose Gaussian $\nu(s) = e^{-bs/2}$ b<0 is realistic

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$$

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{(3/\kappa)} \frac{1}{\nu(s)}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\left(3+\frac{3}{\kappa}\right)}$$

- Ellipsoidal symmetry

$$s = \frac{x^2}{X(t)^2} + \frac{y^2}{Y(t)^2} + \frac{z^2}{Z(t)^2}$$

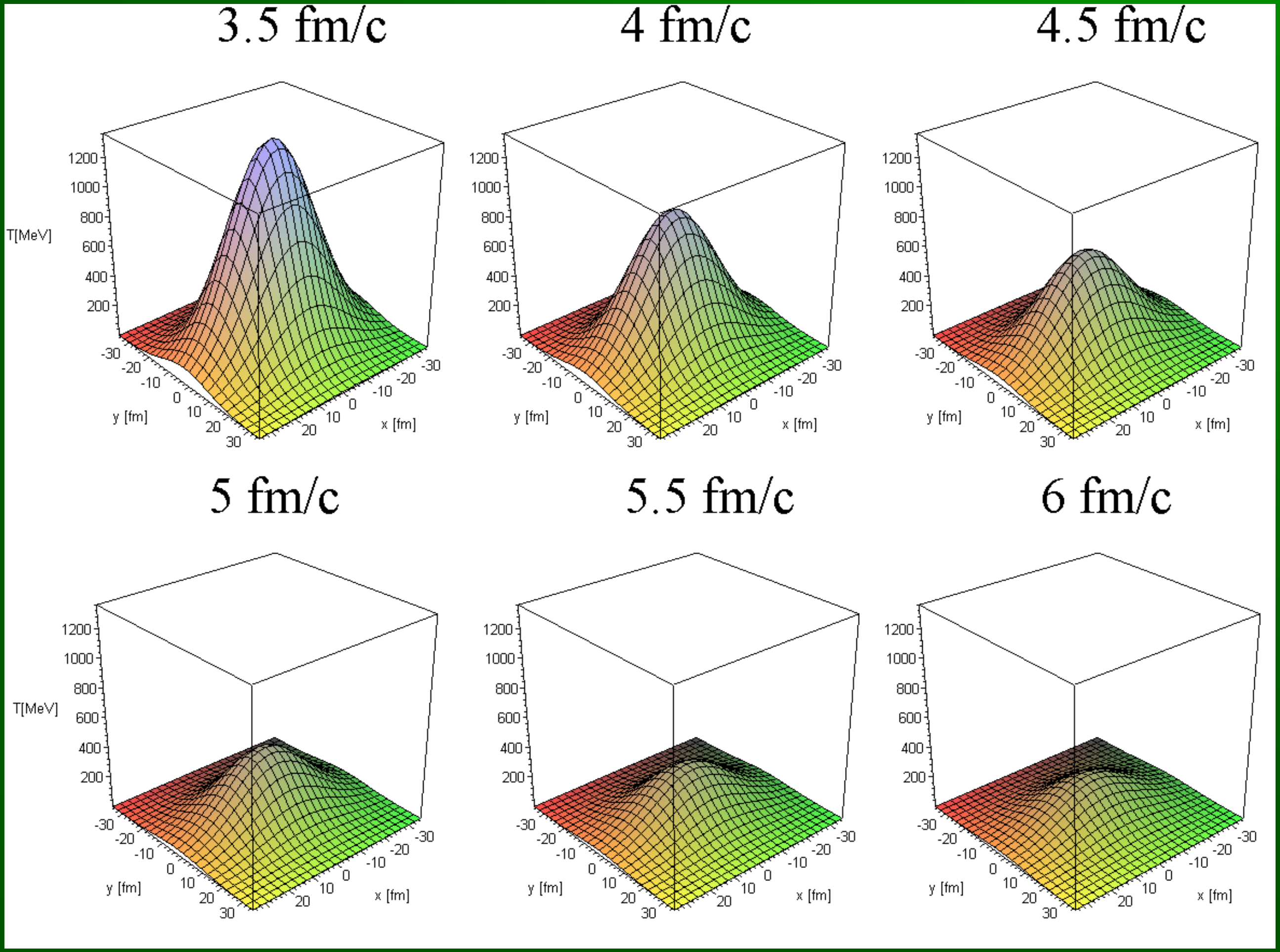
$$u^\mu = \gamma \left(1, \frac{\dot{X}(t)}{X(t)} x, \frac{\dot{Y}(t)}{Y(t)} y, \frac{\dot{Z}(t)}{Z(t)} z\right)$$

  - (thermodynamic quantities const. on the s=const. ellipsoid)
- Directional Hubble-flow
  - v=Hr or H=v/r, the Hubble-constants: $\frac{\dot{X}(t)}{X(t)}, \frac{\dot{Y}(t)}{Y(t)}, \frac{\dot{Z}(t)}{Z(t)}$
  - $\dot{X}(t), \dot{Y}(t), \dot{Z}(t): const.; \ \left(\dot{X}, \dot{Y}\right) \Leftrightarrow \left(u_t, \varepsilon\right)$

(T. Csörgő, L. P. Csernai, Y. Hama és T. Kodama, Heavy Ion Phys. A 21, 73 (2004))

Temperature profile

Transverse temperature profile as a function of time with an example parameter set:



The hadronic source function

- Source function: probability of a particle created at x with momentum p
- Maxwell-Boltzmann distribution + extra terms

$$S(x,p)d^4x \sim n(x)\exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right]\frac{p_\mu u^\mu}{u^0}H(\tau)d^4x$$

  - $H(\tau)d\tau$ freeze-out distribution

    if sudden: $H(\tau)=\delta(\tau-\tau_0)$
  - $\frac{p_\mu u^\mu}{u^0}d^3x$ Cooper-Fry prefactor (flux term)
  - Validity: $\tau_0 > R_{HBT}$, $m_t > T_0$

Hadronic results

- Single particle spectrum

$$N_1(p_t) = \bar{N}\,\bar{V}\left(m_t - \frac{p_t^2(T_{\text{eff}} - T_0)}{m_t T_{\text{eff}}}\right)\exp\left[-\frac{p_t^2}{2m_t T_{\text{eff}}} + \frac{p_t^2}{2m_t T_0} - \frac{m_t}{T_0}\right]$$

$$T_x = T_0 + m_t \dot{X}^2 \frac{T_0}{b(T_0 - E)},\quad T_y = T_0 + m_t \dot{Y}^2 \frac{T_0}{b(T_0 - E)},\quad \frac{1}{T_{\text{eff}}} = \frac{1}{2}\left(\frac{1}{T_x} + \frac{1}{T_y}\right)$$

- Elliptic flow

$$v_2 = \frac{I_1(w)}{I_0(w)} \quad \text{with} \quad w = \frac{p_t^2}{4m_t}\left(\frac{1}{T_y} - \frac{1}{T_x}\right) \sim E_K \frac{\varepsilon}{T_{eff}}$$

, I: Bessel func.

- Two-particle correlation functions: $R_{x,y} = \dfrac{T_0 \tau_0 (T_{x,y} - T_0)}{m_t T_{x,y}}$

$$R_{out} = R_{side} = 0.5\left(R_x^2 + R_y^2\right)$$

Source function of photons

- Photons are continuously created from thermalization
- Photons are not thermalized
- Bose-Einstein distribution

$$S(x,p)d^4x \sim \left(e^{p_\mu u^\mu / T} - 1\right)^{-1} E d^4 x$$

- Has to be integrated over time
- Second order Gaussian approximation

Photon spectra and photon v_2

- Integration can be done analytically

$$N_1(p_t) = (2\pi)^{3/2} p_t R_x R_y R_z \tau_0 \frac{\kappa}{3} A^{-4\kappa/3} \left[A^{3/2}\Gamma\left(-\frac{3}{2}+\frac{4\kappa}{3}, A\xi^{3/\kappa}\right)\Big|_1^{\tau_f/\tau_i} + \frac{B^2}{4} A^{-1/2}\Gamma\left(\frac{1}{2}+\frac{4\kappa}{3}, A\xi^{3/\kappa}\right)\Big|_1^{\tau_f/\tau_i} \right]$$

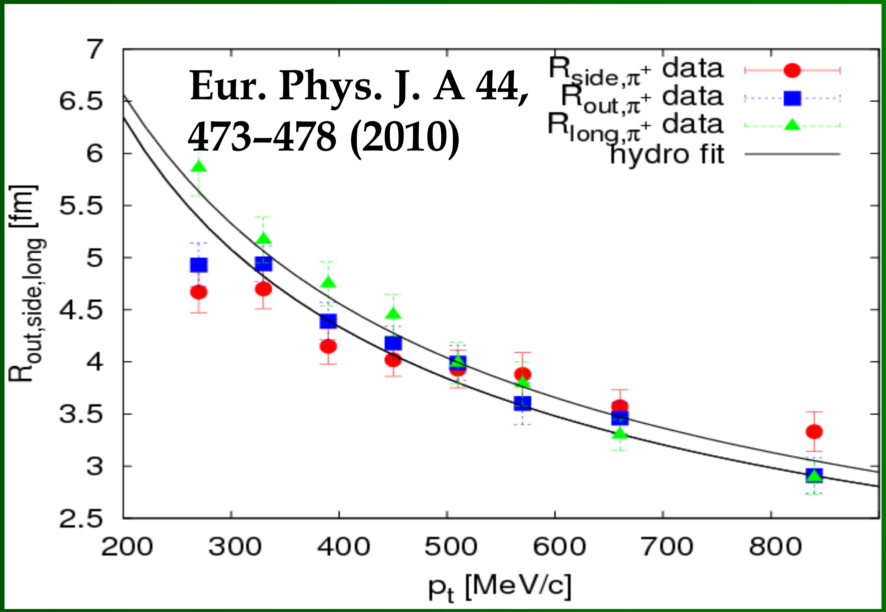
$$v_2 = \frac{B}{2A^2} \frac{\Gamma\left(-\frac{1}{2}+\frac{4\kappa}{3}, A\xi^{3/\kappa}\right)\Big|_1^{\tau_f/\tau_i}}{\Gamma\left(\frac{3}{2}+\frac{4\kappa}{3}, A\xi^{3/\kappa}\right)\Big|_1^{\tau_f/\tau_i}}$$

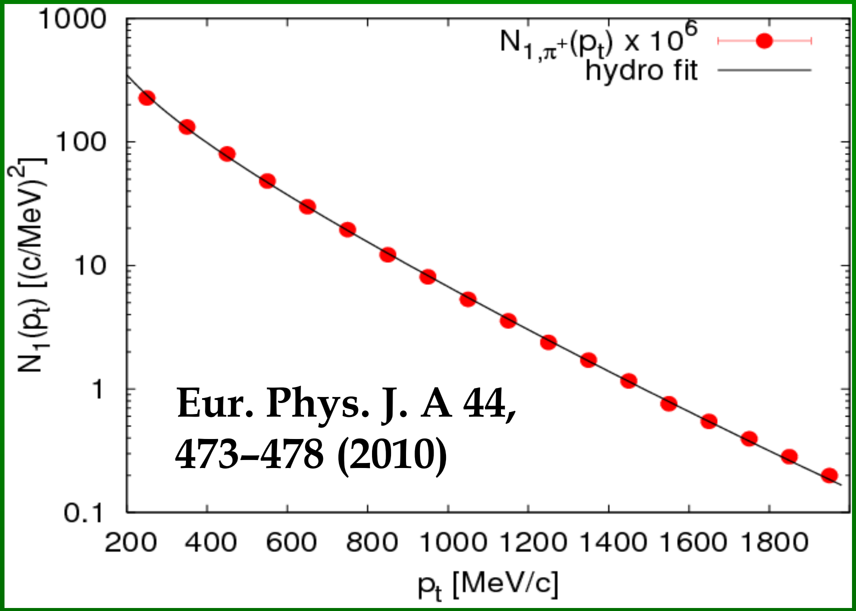
- A and B are:

$$A = \frac{p_t}{T_0}\left[1 - \frac{\kappa(\kappa-3) - \frac{\kappa^2 b}{u_t^2}}{2\left((\kappa-3) - \frac{b}{u_t^2}\right)}\right], \quad B = \frac{p_t}{T_0} \frac{\kappa^2 b \frac{\varepsilon}{u_t^2}}{2\left((\kappa-3) - \frac{b}{u_t^2}\right)}$$

Single pion spectum with HBT radii

- 0-30% centrality, Au+Au, PHENIX

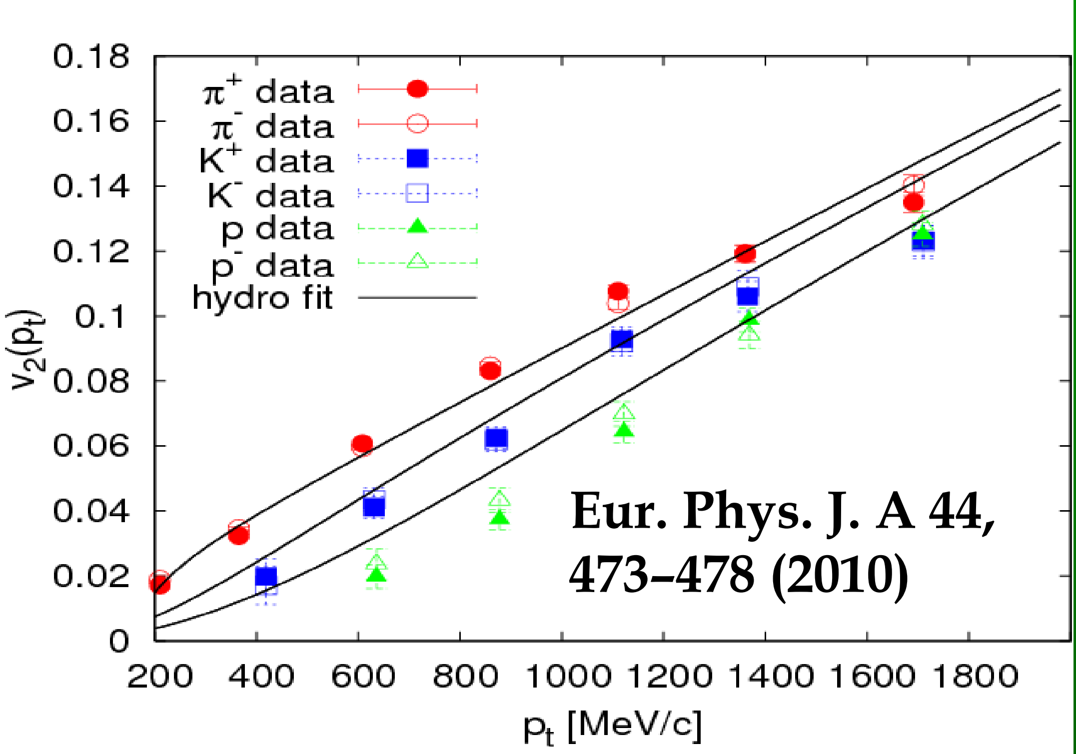




• T_0	199 ± 3 MeV	central freeze-out temp.
• ε	0.80 ± 0.02	momentum space ecc.
• u_t^2/b	-0.84± 0.1 (b<0)	transv. flow/temp. grad
• τ_0	7.7 ± 0.1	freeze-out proper time
• χ^2	171 (24 with theory error)	

Elliptic flow

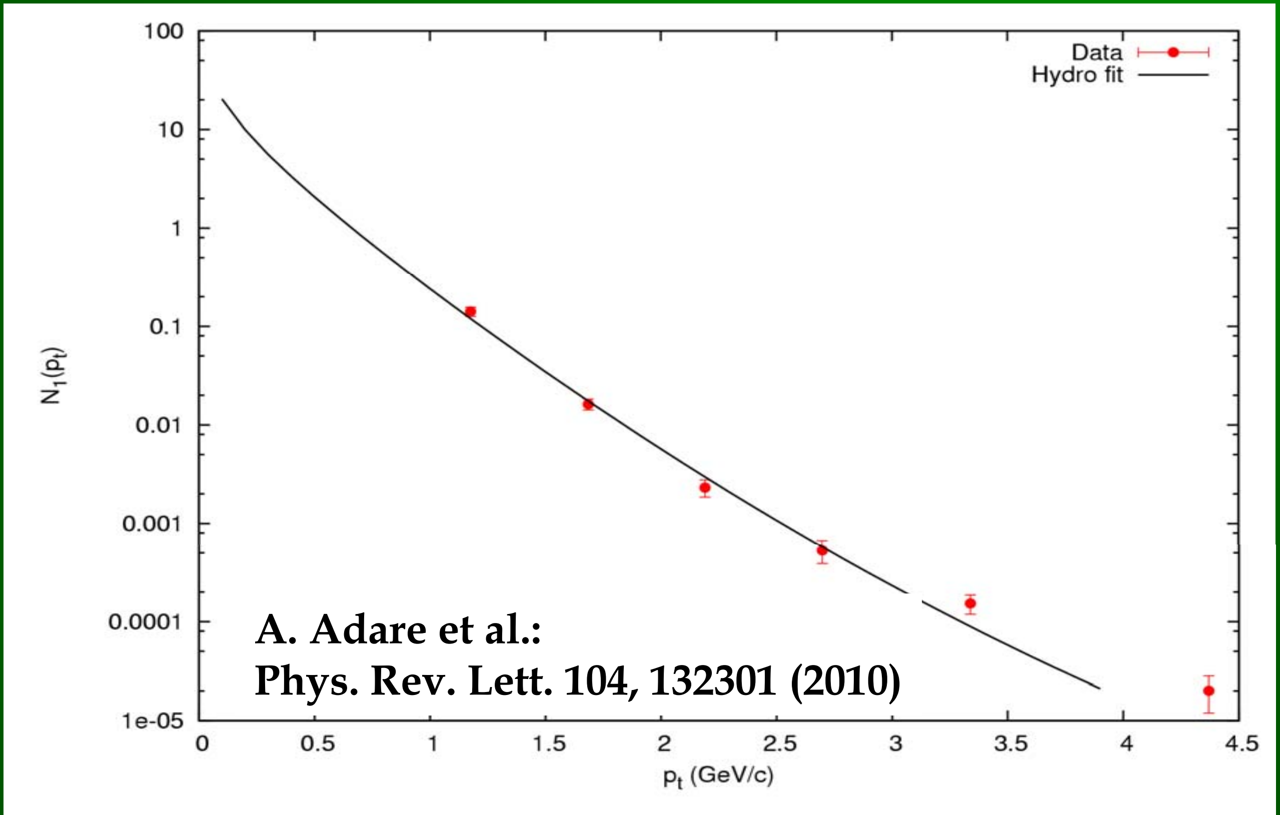
- 0-92% centrality, Au+Au, PHENIX (R.P. method technique)



- T_0 204 ± 7 MeV f.o. temperature
- ε 0.34 ± 0.01 eccentricity
- u_t^2/b -0.34 ± 0.07 (b<0) transv. flow/temp. grad
- χ^2 256 (66 with theory error)

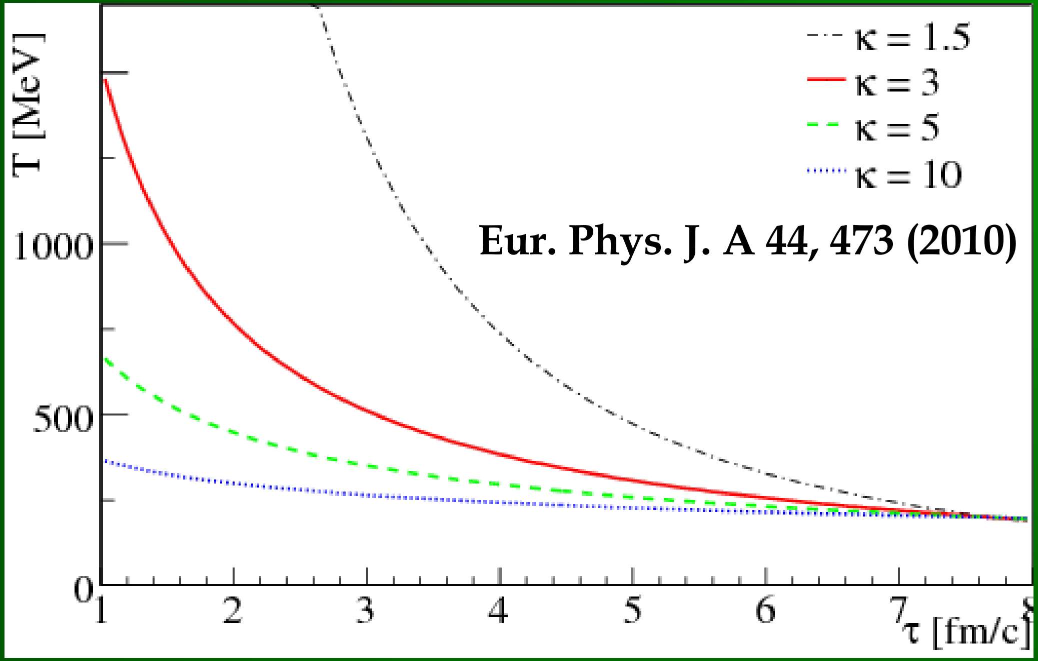
Direct photon spectrum

- Fits to 0-92% centrality PHENIX data
- Parameters from hadronic fit
- Important new parameter: $\kappa=7.7\pm0.8$



Impications on initial temperature

- From hadronic observables:

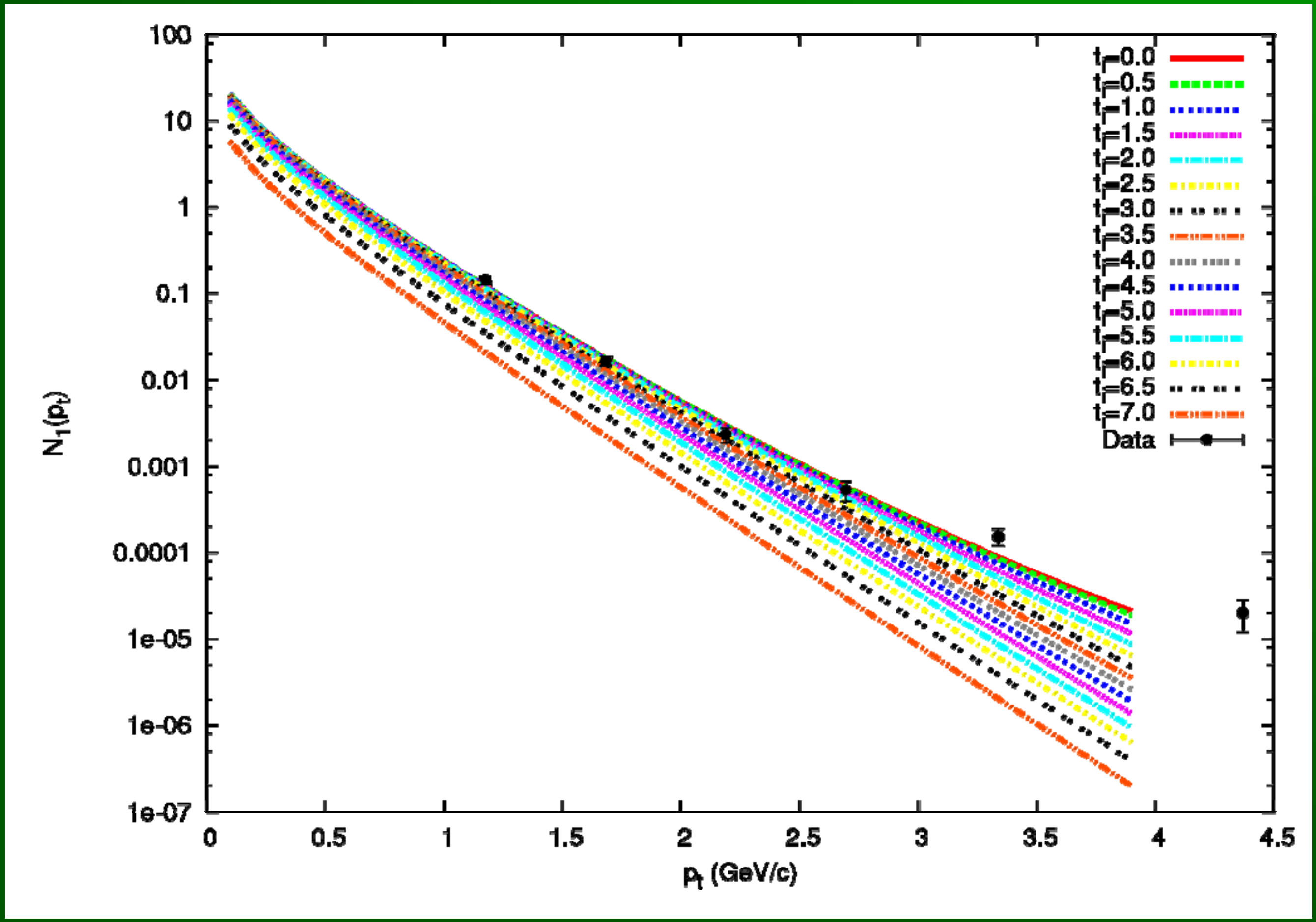


- EoS from photon spectra: κ=7.7±0.8
- Initial temperature (at τ=1 fm/c)

$$T_i \approx 440\ \mathrm{MeV}$$

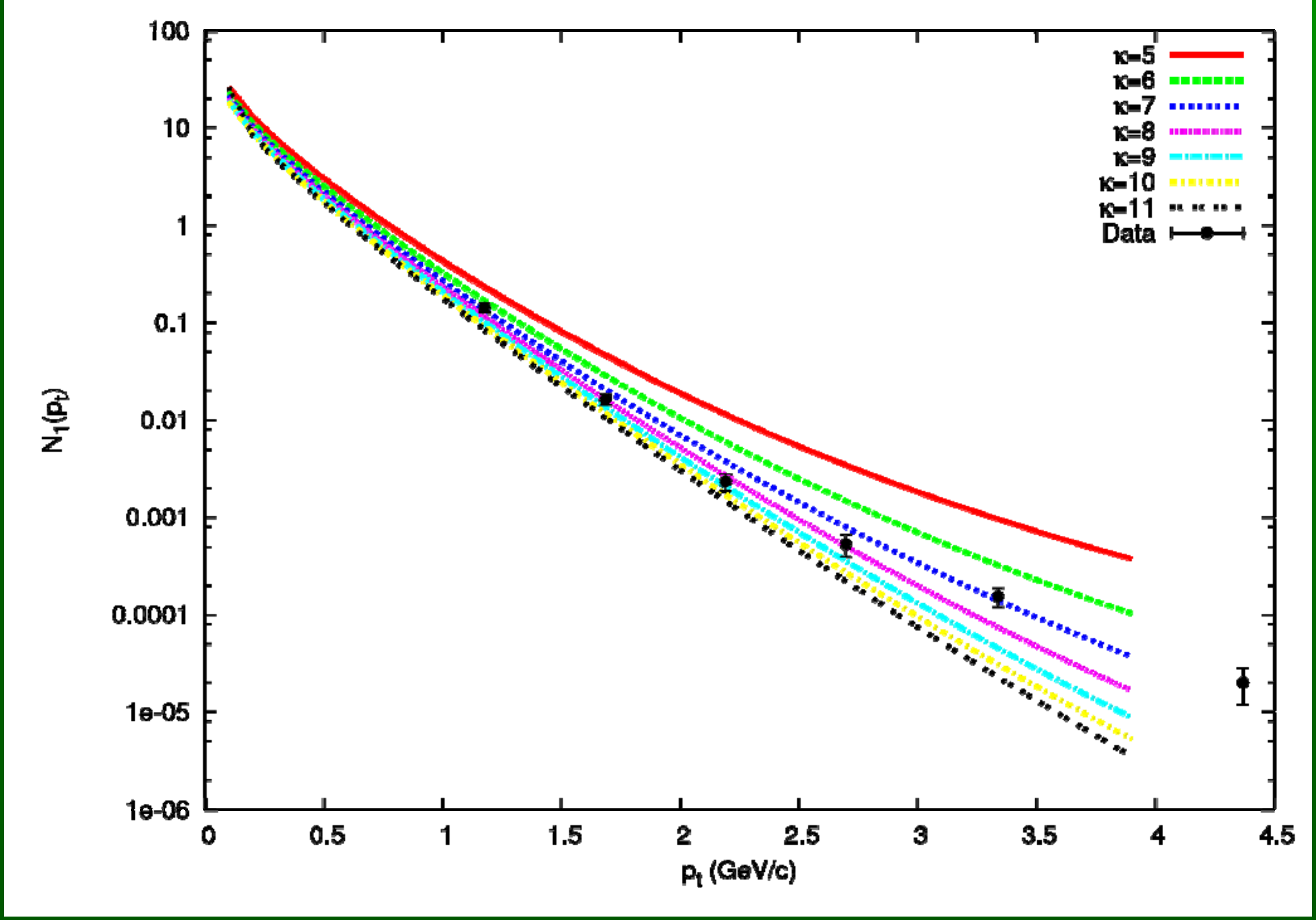
Insensitivity to the initial time

- Initial time period: small contribution

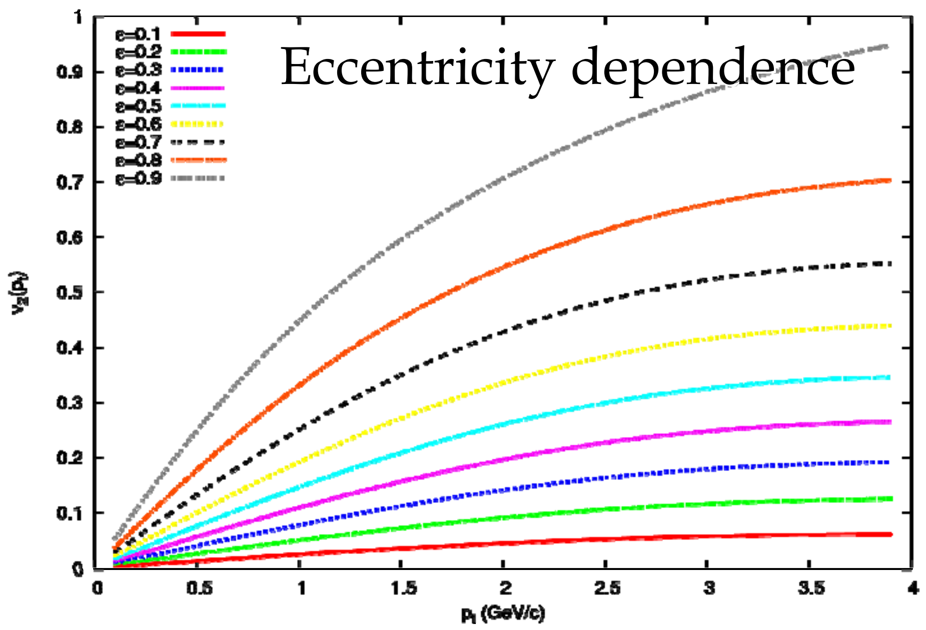


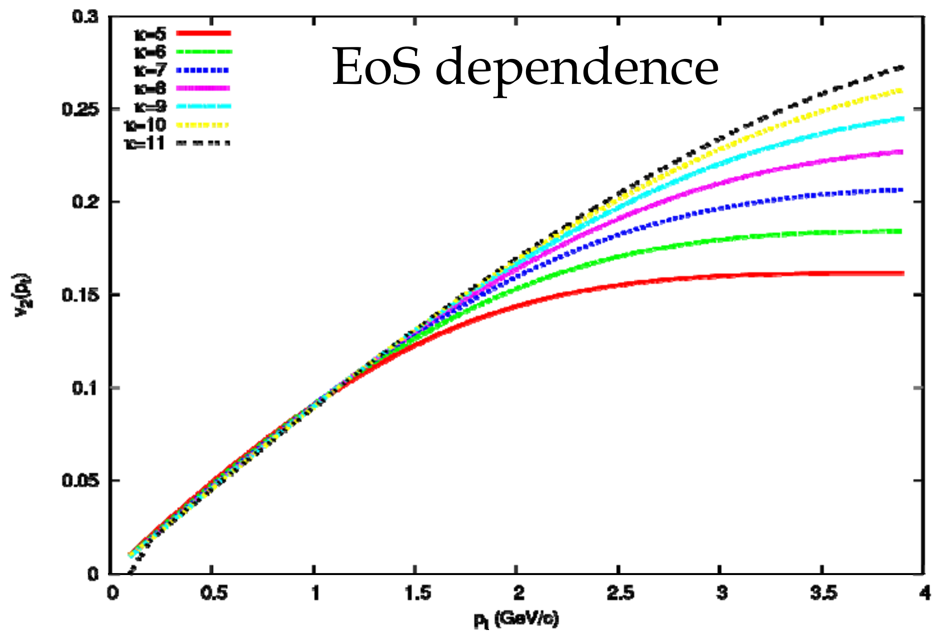
EoS dependence

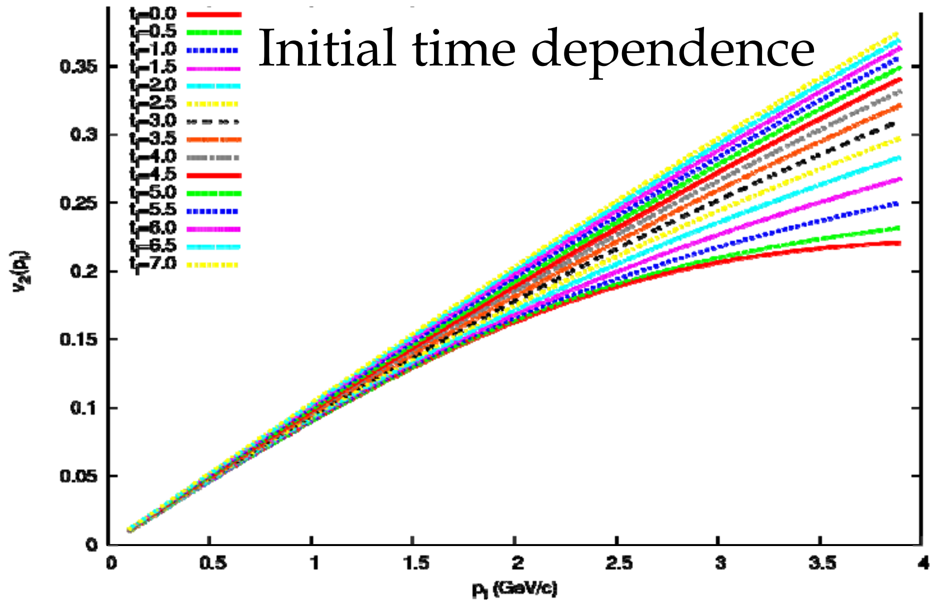
- Sensitive to κ with these level of errors



Photon elliptic flow analysis

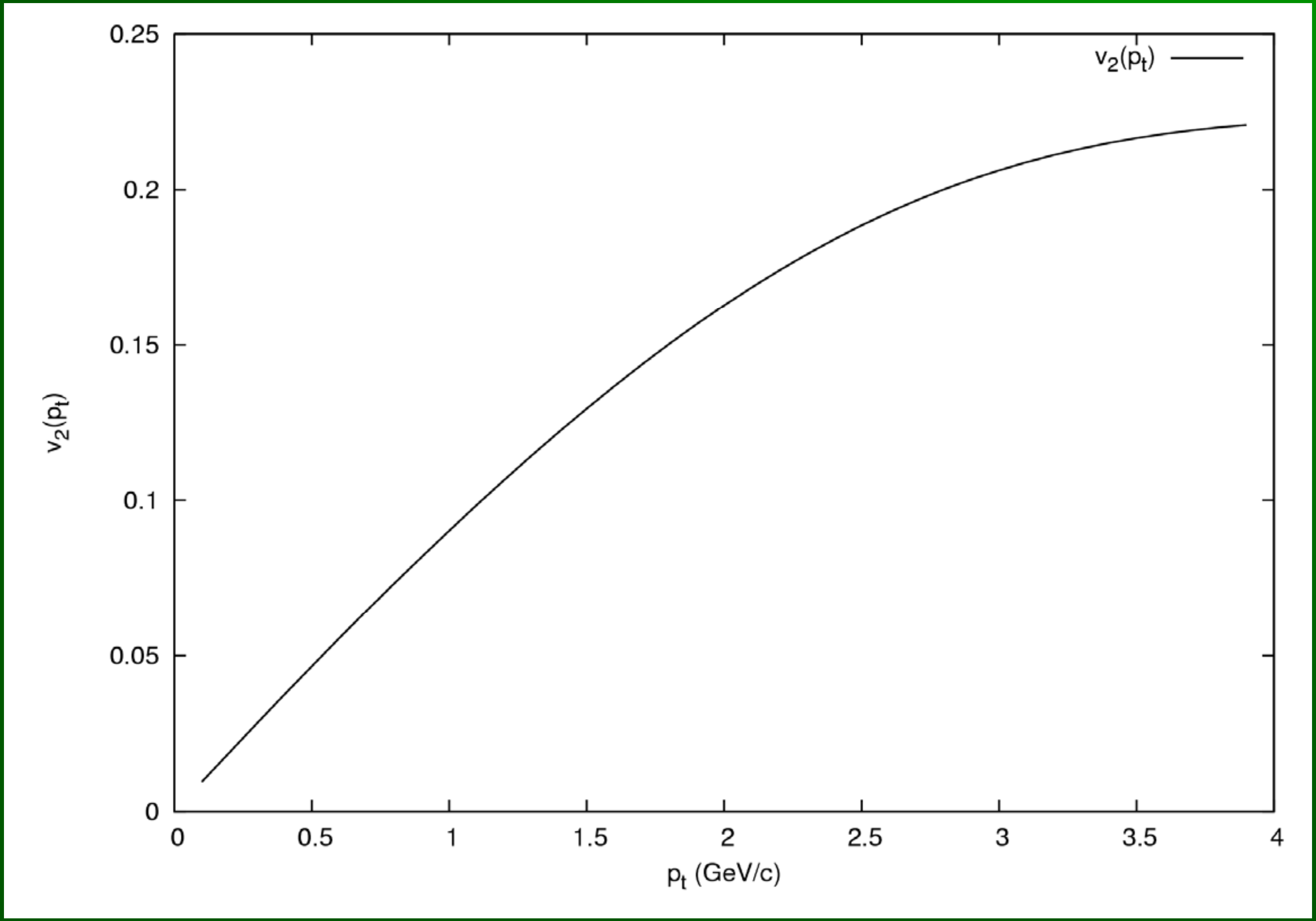






Photon elliptic flow prediction

- With the same parameters, not fine tuned



Summary

- Revival of interest in perfect hydro
- Our model: 3+1d relativistic model w/o acceleration
- Calculated hadronic source → N_1, v_2, HBT
- Calculated photon source → N_1, v_2
- Photon HBT to be calculated yet
- Compared successfully to data, $\kappa=7.7\pm0.8$

  $T_i \approx 440$ MeV

- Predicted photon elliptic flow

Thank you for your attention

Two-particle correlation radii

- Definition: $$C_2(p_1,p_2)=\frac{N_2(p_1,p_2)}{N_1(p_1)N_1(p_1)}$$
- From the source function: $$C_2(q,K)=1+\lambda\left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|^2$$
- Changing coordinates

$$q=p_1-p_2,\ K=0.5\left(p_1+p_2\right)\Rightarrow C_2(q,K)=1+\lambda\exp\left(-R_x^2q_x^2-R_y^2q_y^2-R_z^2q_z^2\right)$$

- Result: $$R_i=\frac{T_0\tau_0(T_i-T_0)}{m_tT_i}$$
- The usual scaling (same for kaons!): $$R_i^2\sim\frac{1}{m_t}$$
- Bertsch-Pratt coordinates: $$R_{out}=R_{side}=0.5\left(R_x^2+R_y^2\right)$$
- Freeze-out: τ = const. $\leftrightarrow$ $\Delta\tau = 0 \rightarrow R_{out} = R_{side}$

Summary of the fit results

- Freeze-out temperature around 200 MeV
- Flow strongly depends on centrality
- Momentum space eccentricity: 0.3-0.9
  - This is flow assymmetry
- Average transverse flow and temp. gradient:
  - Strongly coupled, ratio around 0.3-1.0 (with b<0)
- Confidence levels very low
- With estimated 3% theory error: acceptable

Famous solutions

- Landau's solution (1D, developed for p+p):
  - Accelerating, implicit, complicated, 1D
  - L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51
  - I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529
  - L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309
- Hwa-Bjorken solution:
  - Non-accelerating, explicit, simple, 1D, boost-invariant
  - R.C. Hwa, Phys. Rev. D10, 2260 (1974)
  - J.D. Bjorken, Phys. Rev. D27, 40(1983)
- Others
  - Chiu, Sudarshan and Wang
  - Baym, Friman, Blaizot, Soyeur and Czyz
  - Srivastava, Alam, Chakrabarty, Raha and Sinha

Nonrelativistic solutions

Solution	Symmetry	Density prof.	EoS	Observables
Csizmadia et al. Phys. Lett. B443:21-25, 1998	Sphere	Gaussian	$\varepsilon = \frac{3}{2} nT$	Calculated
Csörgő Central Eur.J.Phys.2: 556-565,2004	Sphere	Arbitrary	$\varepsilon = \frac{3}{2} nT$	Not calculated
Akkelin et al. Phys.Rev. C67,2003	Ellipsoid	Gaussian (T=T(t))	$\varepsilon = \kappa(T) nT$	Calculated
Csörgő Acta Phys.Polon. B37:483-494,2006	Ellipsoid	Arbitrary (T=T(r,t))	$\varepsilon = \kappa nT$	Not calculated
Csörgő, Zimányi Heavy Ion Phys.17:281-293,2003	Ellipsoid	Gaussian	$\varepsilon = \kappa_e nT - B$	Calculated

Where we are

- Revival of interest, new solutions
  - Sinyukov, Karpenko, nucl-th/0505041
  - Pratt, nucl-th/0612010
  - Bialas et al.: Phys.Rev.C76:054901,2007
  - Borsch, Zhdanov: SIGMA 3:116,2007
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  - Mizoguchi et al.: Eur.Phys.J.A40:99-108,2009
  - Beuf et al.:Phys.Rev.C78:064909,2008 (dS/dy as well!)
- Need for solutions that are:
  - accelerating + relativistic+ 3 dimensional
  - explicit + simple + compatible with the data
- Buda-Lund type of solutions: each, but not simultaneously
- Buda-Lund interpolator: hydro inspired source function, interpolates between 3-dimensional B-L solutions: